

Covariance driven subspace methods : input/output vs output-only ¹

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Abstract

Subspace identification algorithms currently emerge as an efficient tool. In this paper, we investigate their use for the input/output identification of the eigenstructure of a linear Multiple Inputs Multiple Outputs (MIMO) system. This situation can be encountered in structural analysis in vibration mechanics, a case which motivated our study. We focus on the following situation : several successive data sets are recorded, with sensors in the structure, and with known input excitations. The interest of this setup is to emulate a situation in which only the outputs of the sensor are available. We first propose a common method for dealing with both situations and then show on real experiments that, if available, input excitations must be taken into account.

1 INTRODUCTION, PRACTICAL MOTIVATIONS

Subspace identification algorithms currently emerge as an efficient tool for many applications ^[1–3]. Our main motivation in this paper is to compare input/output with output-only structural identification in vibration mechanics. This problem consists in identifying the modes and mode shapes of a structure subject to ambient, non measured, vibration excitation on one side and to known input excitations on the other side.

When the vibration and the input excitations are both unknown and not measured, which is most often the case in real world applications ³, one can apply *output-only* identification methods ^[9]. As shown in section 2, subspace methods allow to identify the eigenstructure of a linear MIMO system, which turns out to be equivalent to modes and mode shapes.

We wish now to analyze how subspace methods behave when the following classical setup is used: The structure is subject to ambient, non measured, vibrations on one side and to known input excitations on the other side.

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³Examples are : offshore structures subject to swell, bridges subject to wind and traffic, etc.

This setup is frequently referred to as *input/output* in vibration mechanics ^[6], and we keep this denomination. Of course, one can still directly apply output-only methods, considering that input excitations are a part of the ambient, non measured, vibrations. But the proper way to deal with this case is to take into account the knowledge of the input excitations.

We show in this paper that using the input data can be done with minor modification of the output-only method : since the input excitations are known and statistically independent from the vibrations, the idea is to remove their influence by projecting first the output measurements on the orthogonal space of the input excitations. Then, One can apply output-only methods to these “new measurements”. Unsurprisingly, experiments show that the input/output method performs better than the output-only one. So, when available, input sensors data should be used.

The relevance of the covariance driven subspace identification method in both cases, and its implementation, are the subject of this paper. In section 2 we briefly review the method for output-only identification. The input/output method is stated in section 3. Practical implementation issues are discussed in section 4. In section 5 we compare both methods on real data. Discussion and conclusions follow.

2 SUBSPACE METHODS : QUICK VISIT

General framework As vibration analysis is our motivation, we use structural identification in vibration analysis as our context for discussing modeling issues.

We assume a continuous time linear system of the form :

$$\begin{cases} M\ddot{Z}(s) + C\dot{Z}(s) + KZ(s) = \nu(s) \\ Y(s) = LZ(s) \end{cases} \quad (1)$$
$$E\nu(s)\nu^T(s') = Q\nu\delta(s' - s)$$

where (M, C, K) are the mass, damping, and stiffness matrices, respectively, Z is the state vector (positions or accelerations), Y is the measurement vector (matrix L indicates which components of the state are actually measured), and ν is the

vibration excitation⁴. As reflected by (1), we assume that the ambient excitation is white noise, see [8] for a justification of (or an excuse for) this. The case of non stationary input excitation will be discussed subsequently. Structural identification (e.g., eigenstructure identification) consists in identifying the pairs (μ, ψ_μ) , solutions of

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \quad , \quad \psi_\mu = L\Psi_\mu \quad (2)$$

Now, sampling (1,2) at period δ yields in the usual way the following equivalent discrete time state space form :

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases} \quad e^{\delta\mu} = \lambda, \quad \psi_\mu = \varphi_\lambda \triangleq H\Phi_\lambda \quad (3)$$

where the pairs (λ, Φ_λ) are eigenvalues and eigenvectors of state transition matrix F . Note that the collection of pairs $(\lambda, \varphi_\lambda)$ form a canonical parametrization of the pole part of system (3). As said before, we assume noise (V_k) to be stationary, that its constant covariance matrix is $Q \triangleq \mathbf{E}(V_k V_k^T)$, where T is the transposition operator.

Recall that from

$$R_i \triangleq \mathbf{E} \left(Y_k Y_{k-i}^T \right)$$

subspace identification of eigenstructure $(\lambda, \varphi_\lambda)$ consists of⁵:

$$\mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots & R_p \\ R_1 & R_2 & R_3 & \dots & \vdots \\ R_2 & R_3 & R_4 & \dots & \vdots \\ \vdots & \vdots & \ddots & \dots & \vdots \\ R_p & \vdots & \ddots & \dots & \vdots \end{pmatrix}$$

factorize $\mathcal{H} = \mathcal{O} \mathcal{C}$ where \mathcal{O} and \mathcal{C} are the observability and controllability matrices

$$\begin{aligned} \mathcal{O} &\triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \\ HF^p \end{pmatrix} \\ \mathcal{C} &\triangleq (G \quad FG \quad F^2G \quad \dots \quad F^pG) \\ G &\triangleq \mathbf{E} \left(X_k Y_k^T \right) \end{aligned}$$

$$\text{finally: } \mathcal{O} \longmapsto (H, F) \longmapsto (\lambda, \varphi_\lambda) \quad (4)$$

⁴To simplify the notations, we do not consider additive input in the observation here. Minor modifications are needed for subspace algorithms to take this or measurement noise into account (see Section 4), see also [2] or [7].

⁵We give here a form of subspace algorithm based on output covariance matrices, slightly different from the data driven subspace [2]

Actual implementation The actual implementation is sketched now. We consider the vectors containing the future and past received data, respectively :

$$\mathcal{Y}_k^+ \triangleq \begin{pmatrix} Y_k \\ Y_{k+1} \\ \vdots \\ Y_{k+p} \end{pmatrix}, \quad \mathcal{Y}_k^- \triangleq \begin{pmatrix} Y_k \\ Y_{k-1} \\ \vdots \\ Y_{k-p} \end{pmatrix} \quad (5)$$

Thanks to the stationarity assumption, Hankel matrix writes :

$$(\forall k) \quad \mathcal{H} = \mathbf{E} \left(\mathcal{Y}_k^+ \mathcal{Y}_k^{-T} \right) \quad (6)$$

We are now given a N -size data sample Y_1, \dots, Y_N . From (6), we deduce that the corresponding empirical block-Hankel matrix also writes :

$$\hat{\mathcal{H}} = \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^+ \mathcal{Y}_k^{-T} \quad (7)$$

Then compute the Singular Value Decomposition $\text{SVD}(\hat{\mathcal{H}})$ and truncate the SVD to the desired model order, this yields an estimate $\hat{\mathcal{O}}$ of the observability matrix \mathcal{O} . From $\hat{\mathcal{O}}$, get (\hat{H}, \hat{F}) , and then $(\hat{\lambda}, \hat{\varphi}_\lambda)$.

Remarks

1. So far the model has been introduced with stationary ambient excitation (V_k) . In this case, the consistency of the estimators is a well known result, i.e. the limits of $(\hat{\lambda}, \hat{\varphi}_\lambda)$ are the true parameters $(\lambda, \varphi_\lambda)$ when the size N of the data samples tends to infinity.
2. However, for application to vibration mechanics, it is of interest to consider situations in which the ambient excitation, while still being modeled as white noise, is assumed to be non-stationary, meaning that the covariance matrix $Q_k = \mathbf{E}(V_k V_k^T)$ depends on time index k . Thus, state and observation processes are no longer stationary, so that the matrix $G_k = \mathbf{E}(X_k Y_k^T)$ is time varying, and the same applies for the controllability matrix C_k . Anyway, the consistency of the subspace method described above in such a case has been proved in [9], using an appropriate form of uniform controllability assumption for matrix C_k .
3. If we suppose noisy measurements, the method remains valid under minor modification. The model becomes

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k + W_k \end{cases} \quad (8)$$

where (W_k) is an unmeasured Gaussian white noise with zero mean. It is essential to note that, with this assumption, the measurement noise does not affect the eigenstructure of (8). In order to get rid of this noise, one just

has to shift the Hankel matrix calculus, which becomes :

$$\mathcal{H} \triangleq \begin{pmatrix} R_1 & R_2 & \dots & R_p \\ R_2 & R_3 & \dots & \vdots \\ R_3 & R_4 & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ R_p & \vdots & \dots & \vdots \end{pmatrix}$$

Of course, one can generalize this trick to the case where (W_k) is an $MA(\iota)$ Moving Average Gaussian white noise sequence with zero mean by shifting ι times the Hankel matrix.

3 SUBSPACE IDENTIFICATION FOR INPUT/OUTPUT TIME DATA

Presentation of the model We consider now a state space model with controlled input excitations U_k (what we call input data) of the state variables X_k :

$$\begin{cases} X_{k+1} = FX_k + DU_k + V_k \\ Y_k = HX_k \end{cases} \quad (9)$$

In this case, we have to introduce the vectors containing the past input:

$$\mathcal{U}_k^- \triangleq \begin{pmatrix} U_k \\ U_{k-1} \\ \vdots \\ U_{k-m} \end{pmatrix}$$

$\mathcal{Y}, \mathcal{U}^-$ for the following collections of data

$$\begin{aligned} \mathcal{U}^- &= (\mathcal{U}_1^-, \mathcal{U}_2^-, \dots, \mathcal{U}_N^-) \\ \mathcal{Y} &= (Y_1, Y_2, \dots, Y_N) \end{aligned} \quad (10)$$

where N is the number of samples used.

Recall on projection methods To remove the influence of the input in the output data formulation, we project the output data on the orthogonal space of the past input data, as follows : for two random vectors X and Y with zero means and finite variances, we define as usual the orthogonal projection of X on $sp(Y)$

$$\begin{aligned} X/Y &\triangleq \mathbf{E}(XY^T)\mathbf{E}(YY^T)^\dagger Y \\ X/Y^{ort} &\triangleq X - X/Y \end{aligned} \quad (11)$$

where A^\dagger is the Moore-Penrose pseudo-inverse of A .

In our case, we want to know the projection of each output Y_k on the past inputs U_1, \dots, U_k . Thanks to the property of

contraction of F , one just has to project on the m last inputs U_{k-m}, \dots, U_k , that means on \mathcal{U}_k^- , for m large enough. For this, we need the covariance matrices $\mathbf{E}(Y_k^- \mathcal{U}_k^{-T})$ and $\mathbf{E}(\mathcal{U}_k^- \mathcal{U}_k^{-T})$, which are empirically

$$\begin{aligned} \widehat{C}_o^- &\triangleq \frac{1}{N} \sum_{k=1}^N Y_k \mathcal{U}_k^{-T} \\ \widehat{R}^- &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{U}_k^- \mathcal{U}_k^{-T} \end{aligned} \quad (12)$$

The input/output identification method Now, the orthogonalized outputs Y_k/\mathcal{U}^{ort} are given by

$$Y_k/\mathcal{U}^{ort} = Y_k - \widehat{C}_o^- \widehat{R}^{-\dagger} \mathcal{U}_k^-$$

and with these 'new measures', we can apply the classical output-only identification method as described in section 2 : from the future and past new data $(Y/\mathcal{U}^{ort})_k^+$ and $(Y/\mathcal{U}^{ort})_k^-$, with for example

$$(\mathcal{Y}/\mathcal{U}^{ort})_k^+ \triangleq \begin{pmatrix} Y_k/\mathcal{U}^{ort} \\ \vdots \\ Y_{k+p}/\mathcal{U}^{ort} \end{pmatrix} \quad (13)$$

we can write the new empirical Hankel matrix :

$$\widehat{\mathcal{H}}_{new} = \frac{1}{N} \sum_{k=1}^N (\mathcal{Y}/\mathcal{U}^{ort})_k^+ (\mathcal{Y}/\mathcal{U}^{ort})_k^{-T}$$

So, compare to the output-only case, the modification is minor. Once this new Hankel matrix is computed, the rest of the method is exactly the same as in the output-only case.

Remarks

1. With this method of projection, we need the inputs (U_k) to be stationary, so that the covariance \widehat{R}^- can be empirically estimated. The inputs (U_k) are also required to be statistically independent from the ambient excitation (V_k) (which is always the case in practice).
2. Since we have reduced the input/output problem to the output-only framework, the assumptions on the V_k are the same as before : it is a stationary white noise sequence or a non stationary white noise sequence with an appropriate assumption of controllability. The consistency of the estimators $(\widehat{H}, \widehat{F})$ is then assured.

4 IMPLEMENTATION ISSUES

Vectorized form of the algorithm Let's keep the notations for $p, m, \mathcal{Y}, \mathcal{Y}_k^+, \mathcal{Y}_k^-, \mathcal{U}, \mathcal{U}_k^-$ as above and suppose $m > p$.

Moreover, we introduce the vectors containing the future input data

$$\mathcal{U}_k^+ \triangleq \begin{pmatrix} U_k \\ \vdots \\ U_{k+m} \end{pmatrix},$$

their collections $\mathcal{U}^+ = (\mathcal{U}_1^+, \dots, \mathcal{U}_N^+)$, and in the same way the past and future collections of outputs \mathcal{Y}^- and \mathcal{Y}^+ .

With $m > p$ and considering the vectors \mathcal{Y}_k^+ , \mathcal{U}_k^+ and \mathcal{U}_k^- , an almost equivalent way to remove the influence of the inputs from the outputs is to project the vector \mathcal{Y}_k^+ orthogonally on \mathcal{U}_k^+ and \mathcal{U}_k^- , since for each k

$$(\mathcal{Y}/\mathcal{U}^{ort})_k^+ \approx (\mathcal{Y}/\left(\begin{pmatrix} \mathcal{U}_k^+ \\ \mathcal{U}_k^- \end{pmatrix}\right)^{ort})_k^+$$

We have now to introduce the empirical cross-covariance matrices

$$\begin{aligned} \widehat{C}^{++} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^+ \mathcal{U}_k^{+T} \\ \widehat{C}^{+-} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^+ \mathcal{U}_k^{-T} \\ \widehat{C}^{-+} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^- \mathcal{U}_k^{+T} \\ \widehat{C}^{--} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^- \mathcal{U}_k^{-T}, \end{aligned}$$

and the empirical auto-covariance matrices

$$\begin{aligned} \widehat{R}^{++} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{U}_k^+ \mathcal{U}_k^{+T} \\ \widehat{R}^{+-} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{U}_k^+ \mathcal{U}_k^{-T} \\ \widehat{R}^{--} &\triangleq \frac{1}{N} \sum_{k=1}^N \mathcal{U}_k^- \mathcal{U}_k^{-T} \\ \widehat{R}^{-+} &\triangleq (\widehat{R}^{+-})^T \end{aligned}$$

and, compared to the former empirical Hankel matrix

$$\widehat{\mathcal{H}} = \frac{1}{N} \sum_{k=1}^N \mathcal{Y}_k^+ \mathcal{Y}_k^{-T} = \frac{1}{N} \mathcal{Y}^+ \mathcal{Y}^{-T}$$

the new empirical Hankel matrix

$$\widehat{\mathcal{H}}_{new} \triangleq \frac{1}{N} (\mathcal{Y}^+ / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort}) \mathcal{Y}^{-T}$$

is simply

$$\widehat{\mathcal{H}}_{new} = \widehat{\mathcal{H}} - (\widehat{C}^{++} \widehat{C}^{+-}) \begin{pmatrix} \widehat{R}^{++} & \widehat{R}^{+-} \\ \widehat{R}^{-+} & \widehat{R}^{--} \end{pmatrix}^\dagger \begin{pmatrix} (\widehat{C}^{-+})^T \\ (\widehat{C}^{--})^T \end{pmatrix}$$

That means : once the collections of data \mathcal{Y}^+ , \mathcal{Y}^- , \mathcal{U}^+ and \mathcal{U}^- are constructed, all the algorithm reduces to the multiplication and inversion of some matrices with size comparable to the previous Hankel matrix. The overhead cost is thus very reasonable.

Remark By classical properties of projection, one has also

$$\begin{aligned} \widehat{\mathcal{H}}_{new} &\triangleq \frac{1}{N} \mathcal{Y}^+ (\mathcal{Y}^- / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort})^T \\ \widehat{\mathcal{H}}_{new} &\triangleq \frac{1}{N} (\mathcal{Y}^+ / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort}) (\mathcal{Y}^- / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort})^T \end{aligned}$$

Weighting coefficients Let's come back to the classical output-only identification method of section 2. The principle is to compute the Singular Value Decomposition SVD($\widehat{\mathcal{H}}$) and truncate the SVD to the desired model order, so that $\widehat{\mathcal{H}} \approx U \Delta V^T$. In the Balanced Realization (BR) algorithm, this yields an estimate $\widehat{\mathcal{O}} = U \Delta^{1/2}$ for the observability matrix \mathcal{O} and $\widehat{\mathcal{C}} = \Delta^{1/2} V^T$ for the controllability matrix \mathcal{C} . However, notice that if we compute the SVD of the matrix

$$\widehat{\mathcal{H}}_W = W_1 \widehat{\mathcal{H}} W_2^T$$

with W_1 and W_2 two known invertible matrices, so that $\widehat{\mathcal{H}}_W \approx U_W \Delta_W V_W^T$, the result is the same as previously with $\widehat{\mathcal{O}} = W_1^{-1} U_W \Delta_W^{1/2}$ and $\widehat{\mathcal{C}} = \Delta_W^{1/2} V_W (W_2^T)^{-1}$. A classical approach is to normalize the data such that all singular values lie between 0 and 1 and thus represents the angle between the subspaces of the future and the past data. It also help to avoid numerical problems by equilibrating the effects of the main modes and the ones with the least energy. A usual way to proceed is to take W_1 and W_2 as the inverses of empirical covariances of the future and past output measures, precisely

$$W_1 = (\frac{1}{N} \cdot \mathcal{Y}^+ \mathcal{Y}^{+T})^{-1/2} \text{ and } W_2 = (\frac{1}{N} \cdot \mathcal{Y}^- \mathcal{Y}^{-T})^{-1/2}$$

so that the norm of the weighted empirical Hankel matrix $\widehat{\mathcal{H}}_W$ is equal to 1.

In the framework of stochastic realization and identification, this approach is known as the canonical variate analysis (CVA) and is particularly useful when the order of the process (X_k) is unknown. Adapting this idea to the input-output identification method, where

$$\widehat{\mathcal{H}}_{new} \triangleq \frac{1}{N} (\mathcal{Y}^+ / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort}) \mathcal{Y}^{-T},$$

yields to the weighting matrices W_1 and W_2

$$\begin{aligned} W_1 &= (\frac{1}{N} (\mathcal{Y}^+ / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort}) (\mathcal{Y}^+ / \left(\begin{pmatrix} \mathcal{U}^+ \\ \mathcal{U}^- \end{pmatrix}\right)^{ort})^T)^{-1/2} \\ W_2 &= (\frac{1}{N} \mathcal{Y}^- \mathcal{Y}^{-T})^{-1/2} \end{aligned}$$

Measurement noise If we suppose noisy measurements, for example

$$\begin{cases} X_{k+1} &= F X_k + D U_k + V_k \\ Y_k &= H X_k + W_k \end{cases}$$

one just has to apply the same method as before to the new Hankel matrix \hat{H}_{new} (i.e. shifting) in order to remove the influence of the noise.

5 EXPERIMENTATIONS

The data available in the EUREKA project FLITE are measured for different aircrafts in various in-flight situations and also given by numerical simulation of new aircrafts.

We use here the data supplied by some partners of the FLITE project, the companies Avions Marcel Dassault.

We do the analysis on data corresponding to in-flight measurements; we have successive data sets and so we can examine the evolution of the modal characteristics with the modifications of the aircraft, for example the decreasing of the fuel in the tanks. We only show one experiment for both IO (input/output) and OO (output-only) methods (with BR weightings). The experiment used 12 output sensors and 1 input sensor. Modal analysis was performed using the Scilab Modal toolbox. See [10, 11] for further explanation and experiments on the use and capabilities of this toolbox.

One clearly sees that the stabilization diagrams (frequency / model order) of the IO method (Fig. 1 and Fig. 3) are cleaner, present more stable frequency lines, and even detect some frequencies, which look like spurious modes in the OO method diagram (Fig. 2 and Fig. 4).

Fig. 5 and Fig. 6 show that the damping estimates (damping/ model order) are much more stable in some cases (and at least as good in all cases) for the IO method. Same comments apply for the mode shape estimates (Fig. 7 and Fig. 8) (we display the cross correlation (cosine) of the selected mode shape w.r.t. the other mode shapes in the same frequency line).

6 DISCUSSION

We have presented the input/output method for eigenstructure identification. This type of setup is popular for instance in vibration mechanics. The modal analysis of aircraft structure is a perfect illustration of the need of methods working during the operational period. We have presented the practical capabilities of the subspace identification method for non stationary structures and experiments show that the input/output method performs better in real world applications. So, when available, input sensors data should be used. Nonetheless, the output-only method performs quite similarly in most cases, and should not be avoided when input data are unknown.

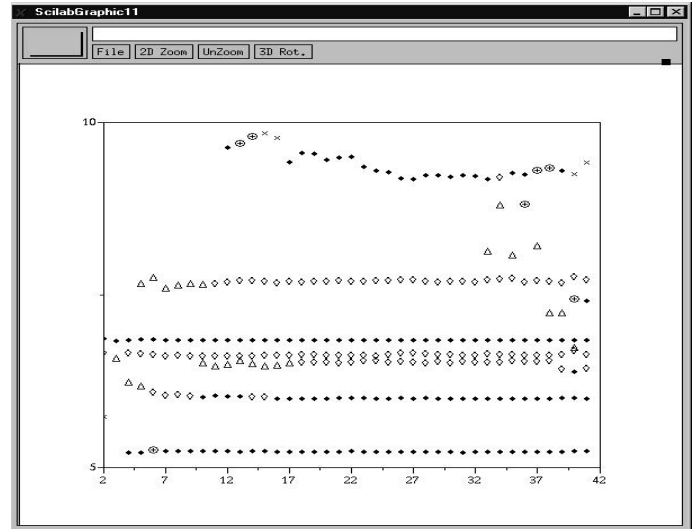


Figure 1: Input/output method : first half of the frequency band

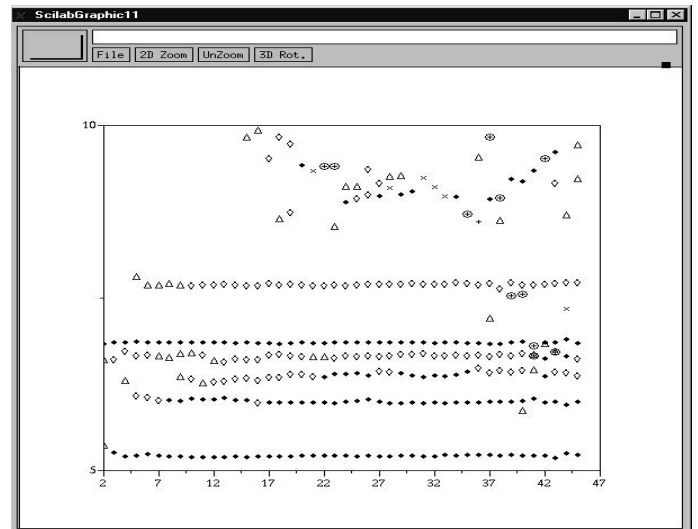


Figure 2: Output-only method : first half of the frequency band

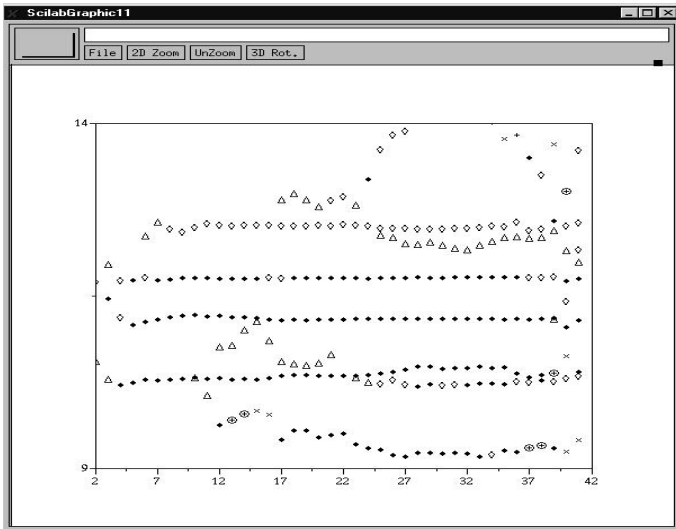


Figure 3: Input/output method : second half of the frequency band

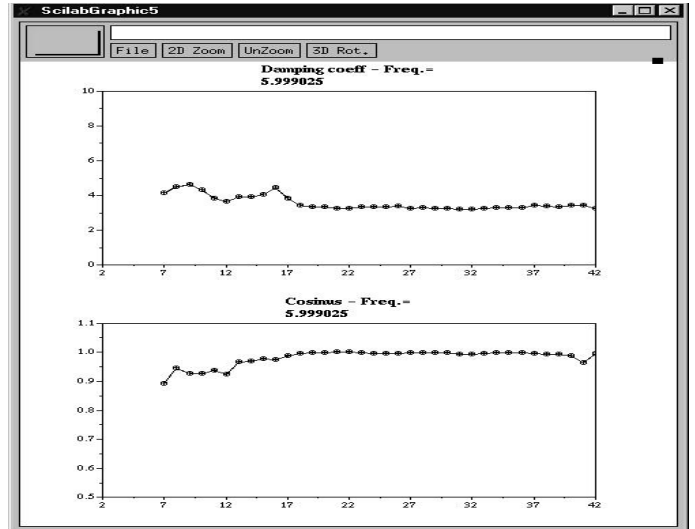


Figure 5: Input/output method : Damping and MAC value of the mode corresponding to frequency 5.99Hz

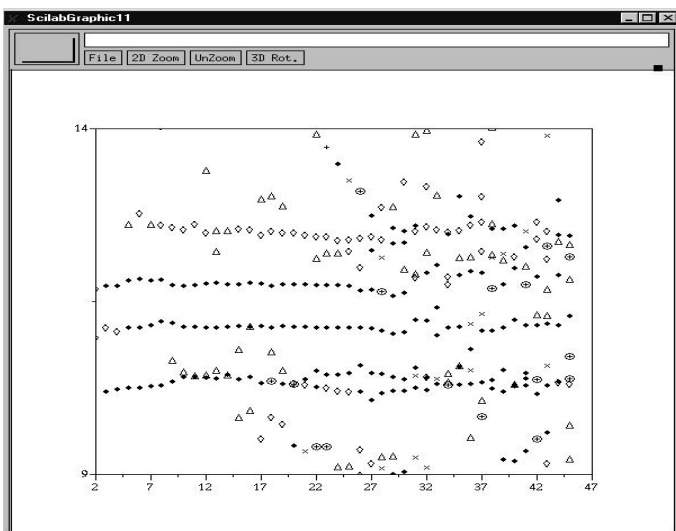


Figure 4: Output-only method : second half of the frequency band

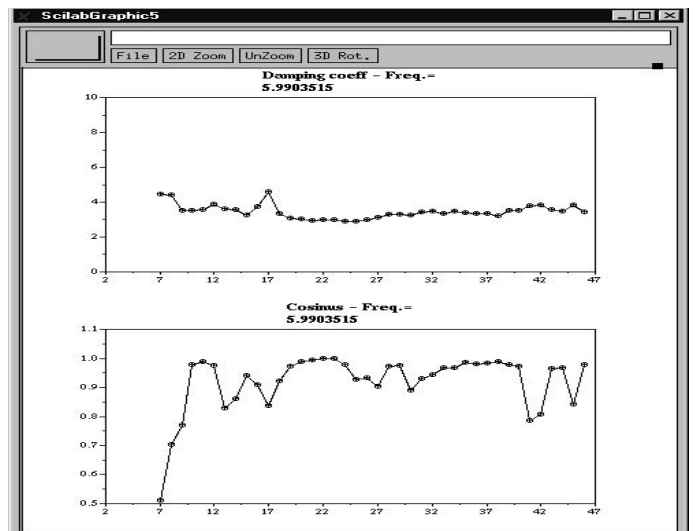


Figure 6: Output-only method : Damping and MAC value of the mode corresponding to frequency 5.99Hz

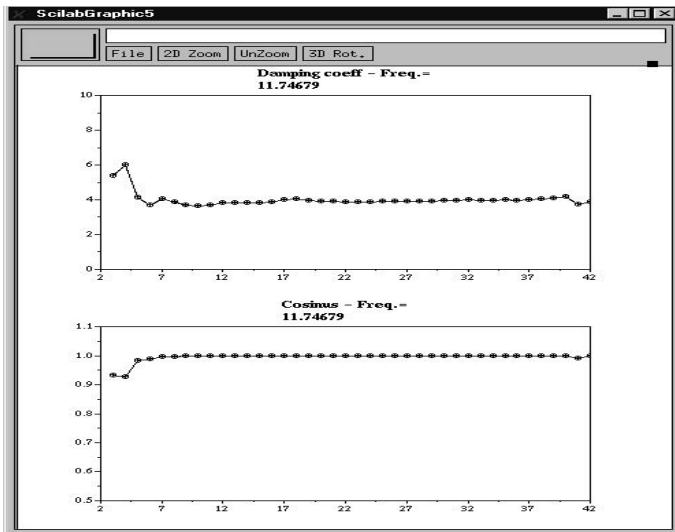


Figure 7: Input/output method : Damping and MAC value of the mode corresponding to frequency 11.75Hz

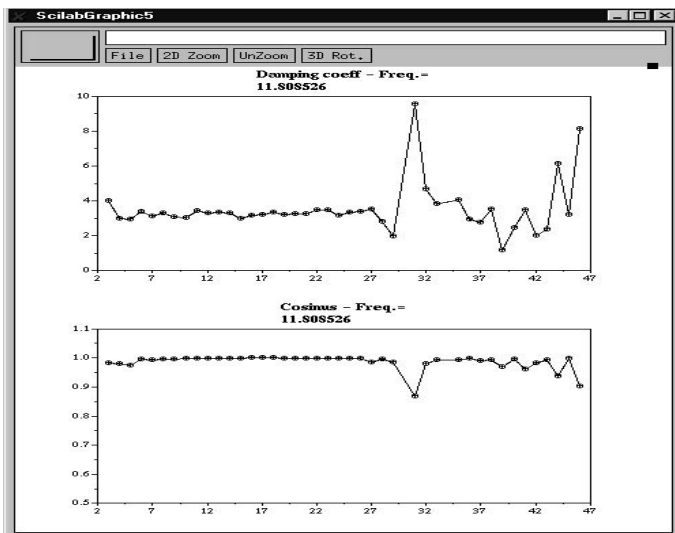


Figure 8: Output-only method : Damping and MAC value of the mode corresponding to frequency 11.81Hz

Since this covariance driven subspace method has been applied to fault detection in the output-only case [7]. We show in future work that the idea presented here can be directly applied to fault detection in the input/output case. All the software needed for this study will be freely available very soon as a Scilab toolbox on the Scilab website.

References

- [1] B. Ottersten, M. Viberg and T. Kailath (1992). Analysis of subspace fitting and ML techniques for parameter estimation. *IEEE Tr. Sign. Proc.*, vol.SP-40, pp. 590-599.
- [2] P. Van Overschee and B. De Moor (1996). *Subspace Identification for Linear Systems: Theory, Implementation, Applications*. Kluwer.
- [3] M. Viberg and B. Ottersten (1991). Sensor array processing based on subspace fitting. *IEEE Tr. Sign. Proc.*, vol.SP-39, no 5, pp.1110-1121.
- [4] L. Hermans and H. Van der Auweraer (1999). Modal testing and analysis of structures under operational conditions: industrial application'. *Mechanical Systems and Signal Processing*, vol.13, no 2, pp.193-216.
- [5] M. Abdelghani, M. Goursat, T. Biolchini, L. Hermans and H. Van der Auweraer (1999). Performance of output-only identification algorithms for modal analysis of aircraft structures. *IMAC-XVII, International Modal Analysis Conference*, Kissimmee, FL.
- [6] F. Deblauwe, R.J. Allemang and D.L. Brown (1987). The polyreference time domain technique, *IMAC-V, International Modal Analysis Conference*, pp. 832-845.
- [7] M. Basseville, M. Abdelghani and A. Benveniste (2000). Subspace-based fault detection algorithms for vibration monitoring. *Automatica*, vol.36, no 1, pp.101-109.
- [8] M. Prevosto, M. Olnon, A. Benveniste, M. Basseville and G. Le Vey (1991). State-space formulation, a solution to modal parameter estimation. *Jal Sound and Vibration*, vol. 148, no 2, pp.329-342.
- [9] A. Benveniste and J-J. Fuchs (1985). Single sample modal identification of a nonstationary stochastic process. *IEEE Tr. Automat. Contr.*, vol.AC-30, pp.66-74.
- [10] M. Goursat, A. Benveniste, M. Basseville and L. Mevel(2000). O2MAD : A SCILAB toolbox for output only modal analysis and diagnostic. *IMAC-XVIII, International Modal Analysis Conference*, San Antonio, Texas.
- [11] M. Goursat, A. Benveniste, M. Basseville and L. Mevel(2002). Using subspace on flight data, a practical example. *IMAC-XXI, International Modal Analysis Conference*, Orlando, Florida.