Errata for *Lectures on the Nearest Neighbor Method*

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- **Page 7, line -8** Replace $B(\mathbf{x}, \rho)$ by $B^\circ(\mathbf{x}, \rho)$ (pointed out by Aditya Ghosh).
- **Page 11, line 1** Replace the first three lines by:

  Note that

  $$\frac{n_{\ell+1}}{n_\ell + 1} \geq 1, \quad \frac{n_{\ell+1}}{n_\ell + 1} \leq \frac{(1+\delta)^{\ell+1}}{(1+\delta)^\ell} = 1 + \delta,$$

  and

  $$\frac{k_{n_{\ell+1}}}{k_{n_\ell + 1}} \leq \psi\left(\frac{n_{\ell+1} - n_\ell - 1}{n_\ell + 1}\right) \leq \psi(\delta).$$

  So,

  $$\frac{n_\ell + 1}{n_{\ell+1}}(1+\varepsilon)k_{n_\ell+1} \geq \frac{(1+\varepsilon)k_{n_{\ell+1}}}{(1+\delta)\psi(\delta)},$$

  and thus, for all δ small enough, by Chernoff's bound... (pointed out by Aditya Ghosh).
- **Page 71, Theorem 6.3** Replace $(\ell+1)$-th by $(\ell+1)k$-th.
- **Page 79, line 12** The formula should be $2\log(n+1) + \gamma$, without the extra v (pointed out by Christian Rau).
- **Page 73, Remark 6.6** "worst" should be "worse".
- **Page 88** Replace the last line by

  $$f(x) \leq \psi(\beta, c) \stackrel{\text{def}}{=} \max\left(\left(\frac{\beta+1}{2\beta}\right)^{\frac{\beta}{\beta+1}} c^{\frac{1}{\beta+1}}, 1 + \frac{2c}{\beta+1}\right)$$

  (pointed out by Jiantao Jiao).
- **Page 89, Lemma 7.2** The correct statement and proof of the lemma are as follows (pointed out by Jiantao Jiao):

Lemma 7.2. *If f is a Lipschitz density on $[0,1]$ satisfying $|f(x) - f(x')| \le c|x-x'|^\beta$ $(x, x' \in [0,1])$ for $c > 0$ and $\beta \in (0,1]$, then*

$$\max_{x\in[0,1]} f(x) \le \psi(\beta, c) \stackrel{\text{def}}{=} \max\left(\left(\frac{\beta+1}{2\beta}\right)^{\frac{\beta}{\beta+1}} c^{\frac{1}{\beta+1}}, 1 + \frac{2c}{\beta+1}\right).$$

Proof. Let $x_0 \in [0,1]$ be such that $f(x_0) = M \stackrel{\text{def}}{=} \max_{x\in[0,1]} f(x)$. Assume that $[x_0 - (\frac{M}{c})^{1/\beta}, x_0 + (\frac{M}{c})^{1/\beta}] \subseteq [0,1]$. Since $f(x) \ge \max(0, M - c|x-x_0|^\beta)$, we have

$$\begin{aligned} 1 = \int_0^1 f(x)\mathrm{d}x &\ge \int_{x_0-(\frac{M}{c})^{1/\beta}}^{x_0+(\frac{M}{c})^{1/\beta}} \left(M - c|x-x_0|^\beta\right)\mathrm{d}x \\ &= \int_{-(\frac{M}{c})^{1/\beta}}^{(\frac{M}{c})^{1/\beta}} \left(M - c|x|^\beta\right)\mathrm{d}x \\ &= 2M\left(\frac{M}{c}\right)^{1/\beta} - \frac{2c}{\beta+1}\left(\frac{M}{c}\right)^{1+\frac{1}{\beta}} \\ &= \frac{2M^{1+\frac{1}{\beta}}}{c^{1/\beta}} \times \frac{\beta}{\beta+1}. \end{aligned}$$

Therefore,

$$M \le \left(\frac{\beta+1}{2\beta}\right)^{\frac{\beta}{\beta+1}} c^{\frac{1}{\beta+1}}.$$

One shows with similar arguments that when $(x_0 - (\frac{M}{c})^{1/\beta} < 0$ and $x_0 + (\frac{M}{c})^{1/\beta} > 1)$

$$M \le 1 + \frac{2c}{\beta+1},$$

while when $(x_0 - (\frac{M}{c})^{1/\beta} < 0$ and $x_0 + (\frac{M}{c})^{1/\beta} \le 1)$ or $(x_0 - (\frac{M}{c})^{1/\beta} \ge 0$ and $x_0 + (\frac{M}{c})^{1/\beta} > 1)$,

$$M \le \left(\frac{\beta+1}{\beta} + \frac{c}{\beta}\right)^{\frac{\beta}{\beta+1}} c^{\frac{1}{\beta+1}}.$$

□

- **Page 161, lines -8 and -7** Replace "under the probability sign" by "outside the probability sign" (pointed out by Christian Rau).
- **Page 237, line 9** $\mathrm{Ber}(\mu(\mathbf{x}))$ should be $\mathrm{Ber}(r(\mathbf{x}))$.

- **Page 242, line 3** "Stone's 1997 paper" should be "Stone's 1977 paper" (pointed out by Christian Rau).

- **Page 261, Theorem 20.9** The bounded difference condition should be

$$\sup_{\substack{(x_1,\ldots,x_n)\in A^n \\ x_i'\in A}} |g(x_1,\ldots,x_n) - g(x_1,\ldots,x_{i-1},x_i',x_{i+1},\ldots,x_n)| \le c_i, \quad 1 \le i \le n,$$

  for some positive constants $c_1,\ldots,c_n$.

- **Page 269, Lemma 20.5** Replace $\mathscr{C}(\mathbf{z},\pi/6)$ by $\mathscr{C}(\mathbf{z},\theta)$ in the second line (pointed out by Aditya Ghosh).

- **Page 271, Remark 20.1** $\int_{\mathbb{R}^d} |g(\mathbf{y})| \log^+ |g(\mathbf{y})| \mathrm{d}\mathbf{y} < \infty$ should be $\int_{\mathbb{R}^d} |g(\mathbf{y})| (\log^+ |g(\mathbf{y})|)^{d-1} \mathrm{d}\mathbf{y} < \infty$ (pointed out by Arnaud Guyader).

- **Page 273, Lemma 20.7** The second statement of the lemma should be: Moreover, for any Borel set $A \subseteq \mathbb{R}^d$,

$$\mu_1\left(\left\{\mathbf{x}\in A : \limsup_{\rho\downarrow 0}\left(\frac{\mu_2\left(B_\rho(\mathbf{x})\right)}{\mu_1\left(B_\rho(\mathbf{x})\right)}\right) > t\right\}\right) \le \frac{c}{t}\,\mu_2(A)$$

  (pointed out by Jiantao Jiao).