In search of a grand unifying theory

Tomasz Bielecki, Areski Cousin, Stéphane Crépey and *Alexander Herbertsson* describe the construction of a model that aims to make credit correlation work bottom-up and top-down

In this article we introduce our work to build a common shock model of portfolio credit risk where one can build a consistent picture of bottom-up defaults that are also manageable in a top-down aggregate loss space. In this sense this model solves the bottom-up top-down puzzle, which the CDO industry had been trying – and failing – to crack since before the crisis.

Two types of shock cause defaults

In our model, defaults are the consequence of shocks associated with groups of obligors. We define the following pre-specified set of groups:

$$y = \{\{1\}, \dots, \{n\}, I_1, \dots, I_m\},$$

where $I_1, \dots, I_m$ are subsets of $N = \{1, \dots, n\}$, and each group I_j contains at least two obligors or more. The shocks are divided in two categories: the idiosyncratic shocks associated with singletons $\{1\}, \dots, \{n\}$ can only trigger the default of name $1, \dots, n$ individually, while the systemic shocks associated with multi-name groups $I_1, \dots, I_m$ may simultaneously trigger the default of all names in these groups. Note that several groups I_j may contain a given name i, so that only the shock occurring first effectively triggers the default of that name.

As a result, when a shock associated with a specific group occurs at time t, it only triggers the default of names that are still alive in that group at time t. In the following, the elements Y of y will be used to designate shocks, and we let $\mathcal{I} = (I_l)_{1 \le l \le m}$ denote the pre-specified set of multi-name groups of obligors.

Shock intensities $\lambda_Y(t, \mathbf{X}_t)$ will be specified later in terms of a Markovian factor process $\mathbf{X}_t$. Letting $\Lambda_t^Y = \int_0^t \lambda_Y(s, \mathbf{X}_s)ds$, we define

$$\tau_Y(t) = \inf\{t > 0; \Lambda_t^Y > E_Y\}, \quad (1)$$

for independent standard exponential random variables E_Y.

For every obligor i we let

$$\tau_i = \min_{\{Y \in y;\, i \in Y\}} \tau_Y(t), \quad (2)$$

which defines the default time of obligor i in the common shocks model. The model filtration is given as $\mathbb{F} = \mathbb{X} \vee \mathbb{H}$, the filtration generated by the factor process $\mathbf{X}$ and the point process $\mathbf{H} = (H^i)_{1 \le i \le n}$ with $H_t^i = \mathbb{1}_{\tau_i \le t}$.

Figure 1: Possible path of defaults

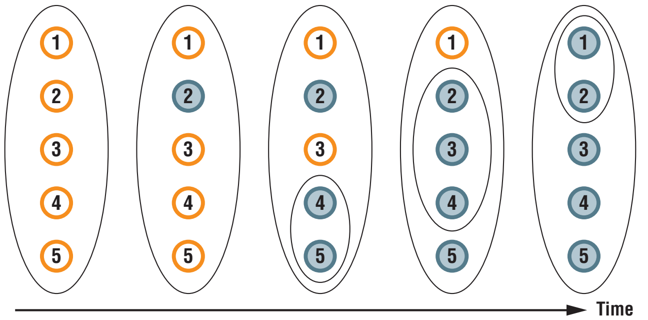


Figure 1: Possible defaults path with n = 5 and y = {{1}, {2}, {3}, {4}, {5}, {4, 5}, {2, 3, 4}, {1, 2}}.

This model can be viewed as a generalisation of the Marshall-Olkin model doubly, that is stochastic (via the stochastic intensities Λ^Y) and dynamised (via the introduction of the filtration $\mathbb{F}$). The purpose of the factor process $\mathbf{X}$ is to more realistically model the diffusive randomness of credit spreads. Figure 1 (above) shows one possible defaults path in our model with $n = 5$ and $y = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{4, 5\}, \{2, 3, 4\}, \{1, 2\}\}$.

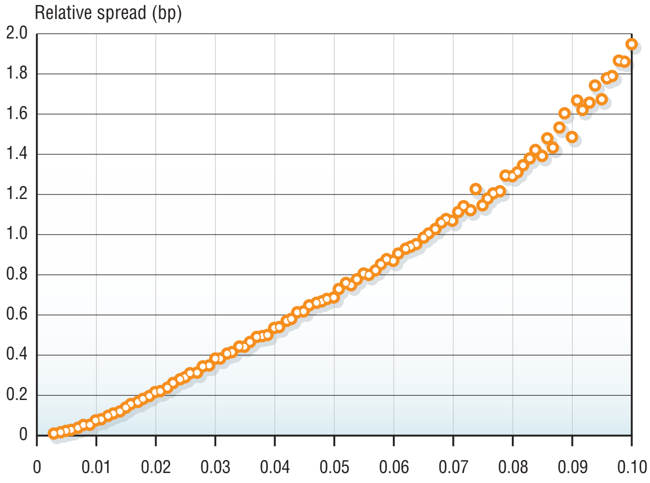
The inner oval shows which common shock happened and caused the observed default scenarios at successive default times. At the first instant, the default of name 2 is observed as the consequence of the idiosyncratic shock $\{2\}$. At the second instant, names 4 and 5 default simultaneously as a consequence of the systemic shock $\{4, 5\}$. At the fourth instant, the systemic shock $\{2, 3, 4\}$ triggers the default of name 3 alone, as names 2 and 4 have already defaulted. At the fifth instant, the default of name 1 alone is observed as the consequence of the systemic shock $\{1, 2\}$.

Demonstrating our calibration results

Tables 1a and 1b (right) summarise the calibration results obtained with this model (using piecewise-constant intensities), for two different quotation dates and two different CDS indices under the constraint that the model perfectly reproduces each individual CDS curve of the corresponding index at these two dates.

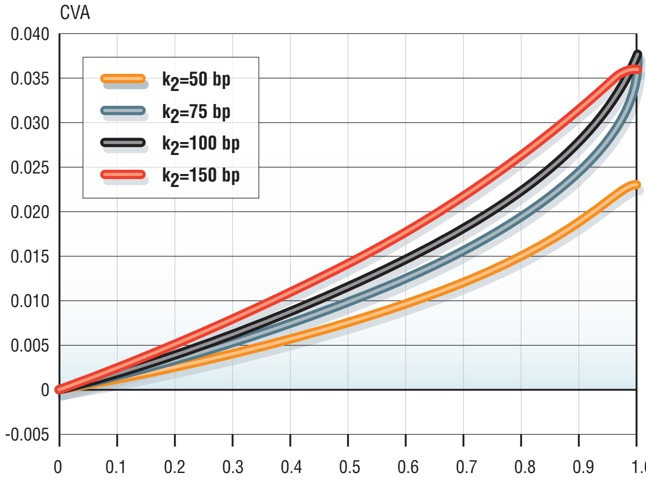
Even better fits can be obtained by resorting to random recovery specifications. The calibrated model can then be used for any bottom-up dynamic portfolio credit purpose, in particular valuation and hedging of counterparty risk on credit derivatives.

Figure 2: Implied volatility of a CDS option



Implied volatility of a CDS option on an individual name with respect to the volatility v of the driving noise of the default intensity.

Figure 3: Time-0 CVA on a CDS with respect to the Gaussian correlation



Time-0 CVA on a CDS with respect to the Gaussian correlation ϱ between the counterparty (protection seller) and the reference firm in a common shock model of the two names.

In this regard, first note that by using suitable stochastic specifications of the shock intensities, the model can generate very significant levels of CDS implied volatilities (see figure 2, above). Figure 3 shows the value of the credit valuation adjustment (CVA) on a payer CDS in the common shock model, with a stochastic default intensity thus specified as a function of a Gaussian copula correlation ϱ between the counterparty (protection seller with credit spread κ_2) and the reference firm of the CDS (with credit spread $\kappa_1 = 84$ basis points).

Changes in the credit valuation adjustment

Observe that the CVA increases monotonically in ϱ, including at the highest values of the latter, whereas co-monotonic pathologies would alter this monotonicity in simplistic models of counterparty credit risk – at least in the case $\kappa_2 = 50\text{bps} < 84\text{bps} = \kappa_1$ (blue curve on the figure) for which, in a co-monotonic model at high ϱ, the reference would always default before the counterparty; hence it would be a zero CVA.

Finally, table 2 (below, left) shows the CVA on stylised $[0-5]\%$, $[5-35]\%$ and $[35-100]\%$ CDO tranches in a common shock model of 100 obligors, including the counterparty of the CDO, without 'naked' and with 'continuous' collateralisation (collateral continuously updated to track at every time the left-limit of the mark-to-market of the CDO tranche, the most extreme case of collateralisation with the left-limit reflecting an 'infinitesimal' cure period).

As is clear from the table, collateralisation has little impact in this case, particularly on the senior tranches, which conveys the important message that due to wrong-way risk (represented in this model by the possibility of joint defaults which are 'missed' by the collateral due to the cure period), it may be difficult to collateralise counterparty risk on credit derivatives.

Table 1a: CDX 2007-12-17

CDO tranche	[0, 3]	[3, 7]	[7, 10]	[10, 15]	[15, 30]
Market spread	48.07	254.0	124.0	61.00	41.00
Model spread	48.07	254.0	124.0	61.00	38.94
Absolute error in bp	0.010	0.000	0.000	0.000	2.061
Relative error in %	0.0001	0.000	0.000	0.000	5.027

Table 1b: iTraxx Europe 2008-03-3

CDO tranche	[0, 3]	[3, 6]	[6, 9]	[9, 12]	[12, 22]
Market spread	40.15	479.5	309.5	215.1	109.4
Model spread	41.68	429.7	309.4	215.1	103.7
Absolute error in bp	153.1	49.81	0.0441	0.0331	5.711
Relative error in %	3.812	10.39	0.0142	0.0154	5.218

Table 1a & 1b: CDX.NA.IG Series 9, 17 December 2007 and iTraxx Europe Series 9, 31 March 2008. The market and model spreads and the corresponding absolute errors, both in basis points and in percent of the market spread. The [0, 3] spread is quoted in %. All maturities are for five years.

Table 2: Naked versus collateralized CVA

	Naked			Collateralised		
Tranche	0-5	5-35	35+	0-5	5-35	35+
CVA	4.78	2.96	2.44	3.41	2.73	2.26

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