XVA Metrics for CCP Optimisation

Claudio Albanese[1], Yannick Armenti[2], and Stéphane Crépey[3]

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Abstract

Based on an XVA analysis of centrally cleared derivative portfolios, we consider two capital and funding issues pertaining to the efficiency of the design of central counterparties (CCPs).

First, we consider an organization of a clearing framework, whereby a CCP would also play the role of a centralized XVA calculator and management center. The default fund contributions would become pure capital at risk of the clearing members, remunerated as such at some hurdle rate, i.e. return-on-equity. Moreover, we challenge the current default fund Cover 2 EMIR sizing rule with a broader risk based approach, relying on a suitable notion of economic capital of a CCP.

Second, we compare the margin valuation adjustments (MVAs) resulting from two different initial margin raising strategies. The first one is unsecured borrowing by the clearing member. As an alternative, the clearing member delegates the posting of its initial margin to a so called specialist lender, which, in case of default of the clearing member, receives back from the CCP the portion of IM unused to cover losses. The alternative strategy results in a significant MVA compression.

A numerical case study shows that the volatility swings of the IM funding expenses can even be the main contributor to an economic capital based default fund of a CCP. This is an illustration of the transfer of counterparty risk into liquidity risk triggered by extensive collateralization.

Keywords: Central counterparty (CCP), initial margin, default fund, cost of funding initial margin (MVA), cost of capital (KVA).

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[1] *Global Valuation ltd, London, United Kingdom*

[2] *BNP Paribas - RISK Department, Grands Moulins de Pantin, 93500, Pantin, France*

[3] *LaMME, Univ Evry, CNRS, Université Paris-Saclay, 91037, Evry, France*

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Email addresses: claudio.albanese@global-valuation.com, yannick.armenti@bnpparibas.com, stephane.crepey@univ-evry.fr

1 Introduction

In the aftermath of the 2008-09 global financial crisis, the banking regulators undertook a number of initiatives to cope with counterparty risk. One major evolution is the generalization and incentivization of central counterparties (CCPs).

A CCP serves as an intermediary during the completion of the transactions between its clearing members, typically the broker subsidiaries of major banks. The portfolio of a CCP clears, i.e., in terms of mark-to-market, the CCP is only an interface between the clearing members. Non-members can have access to the services of a CCP through external accounts by the clearing members, whereas the latter also have direct access to the CCP through so called proprietary accounts. The mandates of the CCP are to centralize the collateralization and settlement of transactions and to rewire or liquidate, in the few days following the default, the CCP portfolio of a defaulted clearing member.

Collateral comes in two forms. The variation margin (VM), which is typically re-hypothecable, tracks the mark-to-market of a portfolio. The initial margin (IM) is an additional layer of margin, typically segregated by the CCP, which is meant as a guarantee against the risk of slippage of a portfolio between default and liquidation. Apart from the variation and initial margin that are also required in bilateral trading (as gradually implemented since September 2016, regarding the IM), the clearing members contribute to a mutualized default fund set against extreme and systemic risk. See Khwaja (2016) for a review of margin and default schemes used by different CCPs on different asset classes.

CCPs are typically siloed into CCP services dedicated to the clearing of specific asset classes. In the case of cash-equity CCP services, the mandate of the CCP reduces to the carry of the settlement risk during the few days between the inception and the settlement of each transaction. In the case of derivative services addressing long-dated products, the margins and the default fund mitigate counterparty risk and the related CVA (credit valuation adjustment). In our CCP setup, the latter is the expected cost triggered by the liquidation of the defaults (if any) of the clearing members. But the margins and the default fund also generate substantial MVA (margin valuation adjustment) and KVA (capital valuation adjustment). These are the respective costs for the clearing members of funding their initial margin and of their capital tied up in the default fund.

This paper bears on CCP derivative services and the related XVA (costs) analysis. See Gregory (2015) for a general XVA reference in book form in the direction of XVA optimisation, encompassing both bilateral and centrally cleared trading. Armenti and Crépey (2017) and Kenyon and Iida (2018) study the cost of a clearing framework for a member of a CCP derivative service, under standard regulatory assumptions on its default fund contribution and assuming the initial margin funded by unsecured borrowing. In the present work we challenge these assumptions in two directions.

First, we point out an organization of a clearing framework, whereby a CCP would also play the role of a centralized XVA calculator and management center. The default fund contributions would become pure capital at risk of the clearing members, remunerated as such at some hurdle rate, i.e. return-on-equity. Moreover, we challenge the current default fund Cover 2 EMIR sizing rule with a broader risk based approach, relying on a suitable notion of economic capital of a CCP. The latter approach was already advocated in Ghamami (2015), but restricted to default losses (as opposed to also IM funding expenses as well as CVA and MVA volatility swings in our paper), in a static setup, and without any numerics.

Second, we assess the efficiency, in a CCP trading setup, of an initial margin lending scheme, whereby a so called specialist lender provides the IM to the CCP on behalf of a clearing member, in exchange of some service fee. In case of default of the clearing member, the specialist lender receives back from the CCP the portion of IM unused to cover losses. We demonstrate, both mathematically and numerically, that margin lending leads to a significant MVA compression with respect to unsecured borrowing. There are several works on the MVA. Green and Kenyon (2015), Anfuso, Aziz, Loukopoulos, and Giltinan (2017), and Antonov, Issakov, and McClelland (2018) bear on an efficient implementation of an MVA computed as per the current regulatory standards for initial margins regarding bilateral or centrally cleared transactions. Andersen, Pykhtin, and Sokol (2017) focus on the residual CVA exposure in the presence of initial margin. Andersen and Dickinson (2019) derive a closed-form representation for the (CVA and the) MVA of a centrally cleared portfolio with locally elliptical distribution. The unrelated MVA issue that we address in this paper is the efficiency of an alternative, specialist lender initial margin lending scheme.

Note that the two proposals discussed in this work have already some incarnations in the industry. An economic capital based sizing rule for the default fund, but with capitalization restricted to default losses as in Ghamami (2015), has been used by the Swiss CCP SIX[1]. Some margin lending industry attempts regarding the variation margin (VM) are commented upon in Albanese, Brigo, and Oertel (2013). However, margin lending is much more difficult to implement for VM than for IM, because fhe former is far larger and more volatile than the latter.

1.1 Outline

The paper is outlined as follows. Section 2 sets out our CCP stage. Section 3 applies the general cost-of-capital principles of Albanese and Crépey (2019, Sections 2 and 3) to the XVA analysis of the derivative portfolio of a clearing member of a CCP designed by current standards. In particular, default fund contributions are, to some extent, non remunerated capital at risk of the clearing members, and raising initial margin can be quite costly. Sections 4 and 5 study ways of compressing the related market inefficiencies. Section 6 extends our analysis to the real-life situation of defaultable end-users and of a bank involved into an arbitrary combination of bilateral and centrally cleared portfolios. Section 7 is a numerical case study in the CCP toy model of Section

[1]See https://www.six-securities-services.com/dam/downloads/clearing/clearing-notices/2017/clr-170420-clearing-notice-margin-en.pdf .

A.

The main results are

- Theorem 4.1, which provides an adaptation to the present setup of Proposition 4.2 for bilateral trade portfolios in Albanese, Crépey, Hoskinson, and Saadeddine (2019). It shows that, under our proposed CCP design, a dynamic and trade incremental cost-of-capital XVA strategy ensures to the clearing members submartingale dividend streams, with drift coefficients corresponding to a hurdle rate h on the capital that they set at risk in the default fund;

- Theorem 5.1, which establishes that resorting to margin lending for its IM is cheaper, for a clearing member bank, than unsecured borrowing;

- The numerical findings of Section 7, which show, in particular, that the volatility of the IM funding expenses can even be the main contributor to an economic capital based default fund of a CCP. This is an illustration of the transfer of counterparty risk into liquidity risk triggered by extensive collateralization: a topic that would deserve more attention and research, also in line with the recommendations made in Cont (2017).

2 Setup

We consider a pricing stochastic basis $(\Omega, \mathbb{G}, \mathbb{Q}^*)$, with model filtration $\mathbb{G} = (\mathfrak{G}_t)_{t\in\mathbb{R}_+}$ and risk-neutral pricing measure $\mathbb{Q}^*$, such that all the processes of interest are $\mathbb{G}$ adapted and all the random times of interest are $\mathbb{G}$ stopping times. The corresponding expectation and conditional expectation are denoted by $\mathbb{E}^*$ and $\mathbb{E}^*_t$. We also introduce the $\mathbb{Q}^*$ value-at-risk and expected shortfall of level a ($\geq 50\%$), $\mathbb{V}\mathrm{a}\mathbb{R}^{*,a}$ and $\mathbb{E}\mathbb{S}^{*,a}$, and their conditional versions $\mathbb{V}\mathrm{a}\mathbb{R}^{*,a}_t$ and $\mathbb{E}\mathbb{S}^{*,a}_t$.

We denote by r a $\mathbb{G}$ progressive OIS (overnight indexed swap) rate process, which is together the best market proxy for a risk-free rate and the rate of remuneration of cash collateral. We write β for the corresponding risk-neutral discount factor.

In real-life a dealer bank is involved with many clients and CCPs, in the context of both centrally cleared and bilateral transactions. Note that, in line with the Volcker rule, a dealer bank is not supposed to do proprietary trading. The proprietary accounts of the clearing members of a CCP are (or should mainly be) used to set up dynamic hedges to bilateral client trades (whereas cleared trades with end-users are back-to-back hedged). For clarity, unless explicitly stated otherwise, we assume that the clearing members do not have proprietary accounts. The latter will be added in Sect. 6.2, along with the question of how to cope with the realistic situation of a bank involved into a combination of bilateral and centrally cleared portfolios.

We consider a CCP with $(n+1)$ risky clearing members, labeled by $i = 0, 1, 2, \ldots, n$. We suppose that each of their default times, τ_i, is endowed with an intensity γ_i (in particular, defaults at any constant or $\mathbb{G}$ predictable time have zero probability). We assume the clearing members perfectly hedged in terms of market risk, so that only the counterparty risk related (including risky funding) cash flows remain. As we exclude proprietary trading, this means that new deals enter the CCP in the form of pairs (or more general packages) of offsetting deals, which are just intermediated by two (or more) clearing members. Admittedly, these assumptions require a not too small number of clearing members, with sufficiently large portfolio and client bases. But these are just the conditions for the good functioning of exchanges in the first place.

The shareholders of a clearing member bank are only impacted by the bank pre-default cash flows. Accordingly, in order to align its prices to the interest of its shareholders, each clearing member is assumed to perform valuation with respect to its survival measure $\mathbb{Q}^i$, with $(\mathbb{G}, \mathbb{Q}^*)$ density process $J^i e^{\int_0^\cdot \gamma^i_s ds}$, where $J^i = \mathbb{1}_{[0,\tau_i)}$ (see Albanese and Crépey (2019, Section 4.1 and before Remark 2.1)). In particular, we have by application of the results of Crépey and Song (2017) (cf. the condition (A) there):

Lemma 2.1 *For every $\mathbb{Q}^i$ (resp. sub-, resp. resp. super-) martingale Y, the process Y stopped before τ_i, i.e. $J^i Y + (1 - J^i) Y_{\tau_i -}$, is a $\mathbb{Q}^*$ (resp. sub-, resp. resp. super-) martingale.* ∎

Remark 2.1 The survival measure formulation is a light presentation, sufficient for the purpose of the present paper, of an underlying reduction of filtration setup, which is detailed in the above-mentioned references (regarding Lemma 2.1, cf. also Collin-Dufresne, Goldstein, and Hugonnier (2004, Lemma 1)). ∎

The procedures used by a CCP for coping with the portfolio of a defaulted member involve a combination of liquidation and auctioning. The latter cannot be modeled realistically at each node of the forward simulation of all risk factors which is required for XVA computations. For simplicity, instead, we assume the existence of a risk-free (hence, non IM or DFC posting) "buffer". This additional clearing member replaces defaulted members in their transactions with the surviving members (or simply puts an end to the contracts that were already with itself), after a liquidation period of length δ (e.g. one week). In the interim, the positions of the defaulted members are carried by the CCP. In particular, the CCP is default-free and therefore assumed to perform risk-neutral valuation, under $\mathbb{Q}^*$ (as explained in Armenti and Crépey (2017, Remark 3.4), the defaultability of the CCP itself is not an essential issue from an XVA point of view).

The default of a major clearing member bank is bound to be more important, in terms of market and systemic impact, than the one of a corporate client. To avoid the introduction of secondary terms, we first assume that end-users are default-free, focusing on the defaultability of the clearing members themselves. The defaultability of the end-users will be added in Sect. 6.1. We denote by

- T: an upper bound on the maturity of all claims in the CCP portfolio (assumed held on a run-off basis until Sect. 3.4), including the liquidation time $\delta > 0$;
- $\bar{t} = t \wedge T$, $t^\delta = t + \delta$, for every $t \geq 0$; in particular, the liquidation time of member i portfolio is τ_i^δ;

- D_t^i: the cumulative contractual cash flow process from the clearing member i to the CCP;

- MtM_t^i: the mark-to-market process of the CCP portfolio, with cash flows D^i, of the member i, i.e. the conditional expectation of its future discounted promised cash flows, computed with respect to its survival measure $\mathbb{Q}^i$;

- $\Delta_t^i = \int_{[t-\delta,t]} \beta_t^{-1}\beta_s dD_s^i$: the cumulative contractual cash flows of the member i, accrued at the OIS rate, accumulated over a past period of length δ;

- $\mathrm{VM}_t^i, \mathrm{IM}_t^i \geq 0, \mathrm{DFC}_t^i \geq 0$: the variation margin (VM), initial margin (IM), and default fund contribution (DFC) of the member i by the CCP at time t.

For simplicity, we assume cash only collateral and default fund contributions, stopped before the default of the related clearing member. All cash accounts accrue at the risk-free rate.

The VM (positive or negative) posted by a clearing member on its client trades is assumed to be exactly offset by the one received on the offsetting deals. Moreover, unless explicitly stated otherwise, we exclude proprietary trading. Hence the FVA (funding valuation adjustment) cost for a member of funding its variation margin is zero in our setup (the FVA issue is postponed to Sect. 6.2, where we add bilateral trading and central clearing proprietary accounts).

Table 1 provides a list of the main financial acronyms used in the paper.

CA	CVA + MVA
CCP	Central counterparty
CVA	Credit valuation adjustment of the reference clearing member $i = 0$
DF(C)	Default fund (contribution)
EC	Economic capital
FTP	Funds transfer price
FVA	Funding valuation adjustment of the reference clearing member $i = 0$
IM	Initial margin
KVA	Capital valuation adjustment
L	Trading loss (and profit) process
MtM	Mark to market
MVA	Margin valuation adjustment
OIS	Overnight index swap
STLOIM	Stress test loss over IM
VM	Variation margin
XVA	Generic "X" valuation adjustment

Table 1: Main Financial Acronyms. ∎

3 XVA Analysis of Current CCP Designs

On the topics of Sect. 3.1, 3.2, and 3.3–3.4 below, see also Gregory (2014), chapters 9, 10, and 13.

3.1 CCP Waterfall Analysis

By current CCP waterfall design standards, in case a member defaults, the resources available to the CCP are used in the following order for coping with the losses triggered by the resolution of the default: first the collateral of the defaulter, then his contribution to the default fund, then the own equity (skin of the game) of the CCP, and eventually the default fund contributions of the survivors, possibly refilled beyond their pre-funded levels, if need be, by the clearing members.

In particular, the DFC of a given member is of an hybrid nature between "defaulter pay" collateral and "survivor pay" capital at risk. Namely, it can be used both for coping with the losses triggered by the liquidation of the clearing member itself, like collateral, and, like capital at risk, of other defaulted clearing members (if their own collateral and DFCs do not suffice). Hence it is only ex post, at the final maturity of the portfolio, that one can determine how much of the DFCs played the role of collateral and how much played the role of capital at risk, for each of the clearing members.

Lemma 3.1 *The loss triggered by the liquidation of the member i at time τ_i^δ is*

$$\Big(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta} - \beta^{-1}_{\tau_i^\delta}\beta_{\tau_i}(\mathrm{VM}^i_{\tau_i-} + \mathrm{IM}^i_{\tau_i-} + \mathrm{DFC}^i_{\tau_i-})\Big)^+. \tag{1}$$

Proof. During the liquidation period $[\tau_i, \tau_i^\delta]$, the CCP substitutes itself to the defaulting member, taking care of all its contractually promised cash flows, which represent a cumulative cost of $\Delta^i_{\tau_i^\delta}$ (including a funding cost at the risk-free rate). At the liquidation time τ_i^δ, the CCP substitutes the buffer to itself as counterparties in all the concerned contracts (see after Remark 2.1), which represents an additional cost $\mathrm{MtM}^i_{\tau_i^\delta}$. Moreover, writing

$$Q^i_{\tau_i^\delta} = \mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta},\ \Gamma^i_{\tau_i^\delta} = \beta^{-1}_{\tau_i^\delta}\beta_{\tau_i}(\mathrm{VM}^i_{\tau_i-} + \mathrm{IM}^i_{\tau_i-} + \mathrm{DFC}^i_{\tau_i-}),\ Q^i_{\tau_i^\delta} - \Gamma^i_{\tau_i^\delta} = \varepsilon_i :$$

- If $Q^i_{\tau_i^\delta} \le \Gamma^i_{\tau_i^\delta}$, then, either $Q^i_{\tau_i^\delta} \le 0$ and an amount $(-Q^i_{\tau_i^\delta})$ is paid by the CCP to the member i (who keeps ownership of all its collateral), or $Q^i_{\tau_i^\delta} \ge 0$ and the ownership of an amount $Q^i_{\tau_i^\delta}$ of collateral is transferred to the CCP. In both cases, the CCP recovers $Q^i_{\tau_i^\delta}$;
- Otherwise, i.e. if the overall collateral Γ^i of the member i does not cover the totality of its debt to the CCP, then, at time τ_i^δ, the ownership of Γ^i is transferred in totality to the CCP.

In conclusion, the loss triggered by the liquidation of the member i is

$$\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta} - \mathbb{1}_{\varepsilon_i>0}\Gamma^i_{\tau_i^\delta} - \mathbb{1}_{\varepsilon_i=0}Q^i_{\tau_i^\delta} = \mathbb{1}_{\varepsilon_i>0}(Q^i_{\tau_i^\delta} - \Gamma^i_{\tau_i^\delta}),$$

which is (1). ∎

3.2 Margin and Default Fund Specifications

In view of (1), a basic IM^i specification, in line with a standard definition of $\mathrm{VM}^i_{\tau_i-}$ as $\mathrm{MtM}^i_{\tau_i-}$, is in the form of a value-at-risk at some level a_{im}, of the δ = one week increment of MtM^i. In practice, the base level of IM required by a CCP is actually a historical value-at-risk, which is unconditional in the sense that very few days reflect market conditions in the aftermath of a bank default. Since initial margin is only ever required in default scenarios, the high quantile levels used for its computation may be misleading and losses above initial margin may easily be understated. In particular, such an IM specification offers no protection against the possibility of further losses in relation to market illiquidity in the aftermath of a major default. CCPs account for the latter by means of IM liquidity add-ons. IM credit add-ons are sometimes also applied to the riskiest clearing members.

The EMIR (European market infrastructure regulation) standard regarding the size of the default fund is Cover 2, i.e. the maximum of the greatest and of the sum between the second and third greatest "stress test losses over IM" (STLOIMs) of its clearing members. Such STLOIMs are computed "under extreme but plausible market conditions" (see European Parliament (2012, article 42, paragraph 3, page 37)). Common orders of magnitude of the default fund are of the order of 10% of the aggregated IM in most CCP derivative services. But they can reach 50% or more of the IM in the case of CCP credit derivative (mainly CDS) services, in order to address the hard wrong-way risk associated with default contagion effects.

The Cover 2 total size of the default fund is allocated between the clearing members into default fund contributions (DFCs) proportionally to their STLOIMs, or sometimes to the IMs themselves. Default fund contributions proportional to the IMs is often felt as "double penalty" by the clearing members with high initial margins. It also yields DFCs proportional to the extent to which the CCP is already protected from each clearing member, instead of what should rather be the opposite (see BIS technical committee of the international organization of securities commissions (2012)). On the other hand, default fund contributions proportional to the STLOIMs makes their evolution particularly unpredictable for the clearing members. In any case, going by either specification, both the size and the allocation of the default fund are purely based on market risk, irrespective of the credit risk of the clearing members. The latter is only accounted for marginally and in a second step, by means of specific credit add-ons to the IM of the riskiest members (see above).

In line with the Cover 2 concept, we do not exclude simultaneous defaults of the clearing members in our model. For any $Z \subseteq \{0, 1, 2, \ldots, n\}$, let τ_Z denote the time when members in Z and only in Z default (or $+\infty$ if this never happens). We conservatively ignore the impact of netting in the context of the joint liquidation of several defaulted members. Hence, in view of (1), at $t = \tau_Z^\delta < T$, the loss triggered by the liquidation of the defaults, which we denote by B_t (for "breach"), is

$$B_t = \sum_{i \in Z} \left(\mathrm{MtM}^i_{\tau_Z^\delta} + \Delta^i_{\tau_Z^\delta} - \beta^{-1}_{\tau_Z^\delta} \beta_{\tau_Z} (\mathrm{VM}^i_{\tau_Z -} + \mathrm{IM}^i_{\tau_Z -} + \mathrm{DFC}^i_{\tau_Z -}) \right)^+. \tag{2}$$

A CCP typically absorbs the most junior tranche of this breach on its own equity process E (dubbed "skin in the game" of the CCP), while the residue,

$$\left(B_{\tau_Z^\delta} - E_{\tau_Z^\delta} \right)^+, \tag{3}$$

is absorbed by the DFCs of the surviving clearing members. However, the skin in the game is typically small and, as already demonstrated numerically Section 8.1 in Armenti and Crépey (2017, arXiv v1 version), negligible from a loss-absorbing, hence XVA, point of view. Given our XVA focus in this paper, in what follows we just set $E = 0$.

3.3 Clearing Member XVA Analysis

Centrally cleared derivative trading comes at the cost of the clearing framework for the clearing members. In the reminder of this section, we apply the incomplete market, XVA approach of Albanese and Crépey (2019, Sections 2 and 3) to the corresponding cost analysis. For notational simplicity we remove any index 0 referring to the reference clearing member 0 (the one we are performing the XVA analysis of). In particular, we rewrite $\tau_0 = \tau$ and $\mathbb{Q}^0 = \mathbb{Q}$ (the survival measure of the clearing member), with associated conditional expectation denoted by $\mathbb{E}_t$.

In view of (3) with $E = 0$ there, at each $t = \tau_Z^\delta < T$, the loss of the reference clearing member, assumed instantaneously realized as refill to its default fund contribution, is

$$\epsilon_{\tau_Z^\delta} = \mu_{\tau_Z^\delta} B_{\tau_Z^\delta} = \mu_{\tau_Z^\delta} \sum_{i \in Z} \left(\mathrm{MtM}^i_{\tau_Z^\delta} + \Delta^i_{\tau_Z^\delta} - \beta^{-1}_{\tau_Z^\delta} \beta_{\tau_Z} (\mathrm{VM}^i_{\tau_Z -} + \mathrm{IM}^i_{\tau_Z -} + \mathrm{DFC}^i_{\tau_Z -}) \right)^+, \tag{4}$$

where μ is a suitable refill allocation weight process. A typical specification of the latter is proportional to the default fund contributions of the surviving members, hence, as long as the reference clearing member is nondefault,

$$\mu = \frac{\mathrm{DFC}}{\mathrm{DFC} + \sum_{i \neq 0} J^i \mathrm{DFC}^i}. \tag{5}$$

Remark 3.1 In practice, there is a cap on the cumulative refills to the default fund by the clearing members, beyond which the CCP defaults. As argued in Armenti and Crépey (2017, Remark 3.4), this is also negligible in the context of XVA analysis. ∎

Under the cost-of-capital XVA approach of Albanese and Crépey (2019, Sections 2 and 3), future counterparty default losses and funding expenses are charged to bank clients at a level making the trading loss and profit of the bank shareholders a risk-neutral ($\mathbb{Q}^*$) martingale. Then an additional KVA premium turns their dividend process into a $\mathbb{Q}^*$ submartingale, with a drift corresponding to a target hurdle rate (i.e., essentially, dividend rate, cf. Theorem 4.1 and its proof) h on their capital at risk.

We suppose that the reference clearing member (playing the role of the bank in the above paragraph) can invest cash at the OIS rate r and borrow collateral to post as IM at a possibly blended spread $\lambda \leq \bar{\lambda}$ over r (cf. Sect. 5), where $\bar{\lambda}$ is its unsecured borrowing spread.

Assuming the integrability required for giving a sense to the right-hand sides in (6) (and recalling (4) for the definition of ϵ):

Definition 3.1 Let $\mathrm{CA} = \mathrm{CVA} + \mathrm{MVA}$, where, for $t < \tau$,

$$\begin{aligned} \mathrm{CVA}_t &= \mathbb{E}_t \sum_{t<\tau_Z^\delta \leq T} \beta_t^{-1} \beta_{\tau_Z^\delta} \epsilon_{\tau_Z^\delta}, \\ \mathrm{MVA}_t &= \mathbb{E}_t \int_t^T \beta_t^{-1} \beta_s \lambda_s \mathrm{IM}_s ds. \ \blacksquare \end{aligned} \tag{6}$$

We denote by L the trading loss (and profit) process of the reference clearing member resulting from its counterparty credit losses and risky funding expenses, and of the fluctuations of its ensuing CVA and MVA liabilities (recall that the FVA of a clearing member bank vanishes in the absence of proprietary trading). We write $\boldsymbol{\delta}_\eta(dt) = d\mathbb{1}_{\{\eta \leq t\}}$ for the Dirac measure at a random time η.

Lemma 3.2 *The trading loss process L is a $\mathbb{Q}$ martingale such that*

$$\begin{aligned} & L_0 = 0 \ \textit{and, for } t < \tau, \\ & dL_t = d\mathrm{CA}_t - r_t \mathrm{CA}_t dt + J_t \sum_Z \epsilon_{\tau_Z^\delta} \boldsymbol{\delta}_{\tau_Z^\delta}(dt) + \lambda_t \mathrm{IM}_t dt. \end{aligned} \tag{7}$$

The reference clearing member shareholder trading loss (and profit), i.e. L stopped before τ (see before Lemma 2.1), is a $\mathbb{Q}^*$ martingale.

Proof. Because the clearing members are assumed to be perfectly hedged in terms of market risk (see Sect. 2), only the counterparty risk related cash flows remain. Hence $L_0 = 0$ and, for $t < \tau$,

$$\begin{aligned} dL_t = & \underbrace{\sum_Z \epsilon_{\tau_Z^\delta} \boldsymbol{\delta}_{\tau_Z^\delta}(dt)}_{\text{Counterparty default losses}} + \underbrace{(r_t + \lambda_t)\mathrm{IM}_t dt}_{\text{IM funding costs}} \\ & - \underbrace{r_t(\mathrm{IM}_t + \mathrm{CA}_t)dt}_{\text{IM and reserve capital accrue at the OIS rate}} + \underbrace{d\mathrm{CA}_t}_{\text{Appreciation of the liability CA}}, \end{aligned}$$

which is (7). The stated martingale properties follow by Definition 3.1 and Lemma 2.1 (applied for $i = 0$). ∎

Via mark-to-model, CA = CVA + MVA is also the amount on the so-called reserve capital account of the clearing member, used by the latter for coping with its expected counterparty default losses and funding expenditures.

As opposed to market risk, counterparty (jump-to-default) risk can hardly be hedged by a bank. Hence the regulator requires that a clearing member bank sets capital at risk, in the form of its DFC (see Sect. 3.1 and cf. also Remark 4.1 later below), in order to deal with the related losses (beyond the expected levels of losses already taken care of by its reserve capital). The shareholders of a bank then want to be remunerated at some hurdle rate h on their capital at risk in the default fund. The corresponding KVA charge is (or would be, cf. Sect. 3.5 and 4.1) sourced from clients at trades inception and deposited on a risk margin account of the bank, from where it is gradually released, at a rate h which is a managerial decision of the bank, into the shareholders' dividend stream.

Accordingly, assuming the integrability of DFC required for giving a sense to the right-hand side in (8):

Definition 3.2 Given a constant hurdle rate h, the KVA of the clearing member is given, for $t < \tau$, by

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^T \beta_t^{-1}\beta_s \mathrm{DFC}_s ds. \blacksquare \tag{8}$$

The connection between this technical specification and the above KVA rationale will be explained by Theorem 4.1 and the ensuing comments.

Remark 3.2 Capital and cost of capital calculations are meant to be performed under the historical probability measure $\mathbb{P}$. But $\mathbb{P}$ is hardly estimable for the purpose of cost of capital calculations, which involve projections over decades in the future. As a consequence, we do all our price and risk computations under the risk-neutral measure $\mathbb{Q}^*$ (or the corresponding survival measures of the clearing members). In other words, as common in XVA computations, we work under the modeling assumption that $\mathbb{P} = \mathbb{Q}^*$, which is calibrated to market quotations of derivative instruments, and we leave the residual uncertainty about $\mathbb{P}$ to model risk.

We emphasize that we calibrate $\mathbb{Q}^*$ to derivative market prices, including the corresponding credit premia, and then we perform all our (including capital) calculations based on $\mathbb{Q}^*$. If we were able to estimate risk premia reliably over time horizons that, in the XVA context, can go decades into the future, then we could have a more sophisticated, hybrid $\mathbb{P} \neq \mathbb{Q}^*$ approach. In the absence of such a reliable methodology, we do it all under $\mathbb{Q}^*$. As, in particular, implied CDS spreads are typically larger than statistical estimates of default probabilities, we believe that this approach is conservative. Moreover, our hurdle rate h can be interpreted as a risk aversion parameter of the clearing member and the ensuing KVA as a corresponding risk premium: see Albanese et al. (2019, Section 3.2). $\blacksquare$

3.4 Funds Transfer Price

The portfolio-wide, all-inclusive XVA of the reference clearing member is the sum between its risk-neutral CA and its KVA risk premium. Accordingly, the all-inclusive incremental XVA, or funds transfer price (FTP), required by the reference clearing member for a new (package of) deal(s) with end-users is

$$\mathrm{FTP} = \Delta\mathrm{CA} + \Delta\mathrm{KVA} = \Delta\mathrm{CVA} + \Delta\mathrm{MVA} + \Delta\mathrm{KVA}. \tag{9}$$

The notation $\Delta\cdot$ in (9) refers to the difference between the portfolio-wide XVAs with and without the new deal, the portfolio being assumed held on a run-off basis (as in the above sections) in both cases. See Theorem 4.1 below for the precise economic justification of an FTP as per (9).

As each clearing member $i = 0, \ldots, n$ has an analogous FTP, the total aggregated counterparty risk add-on that needs be sourced from end-users for the new deals is

$$\mathrm{FTP}^{ccp} = \sum_{i=0}^{n} J^i \mathrm{FTP}^i. \tag{10}$$

Now, a vexing modeling issue is that CVA, MVA, and KVA as per (6) and (8) cannot be computed by the members themselves, by lack of knowledge on other clearing members' positions and on the DFC (sometimes even IM) model used by the CCP. Possible approaches to circumvent this limitation are addressed in Barker, Dickinson, Lipton, and Virmani (2017), Andersen and Dickinson (2019), and Arnsdorf (2012, 2019).

3.5 Current Uses of the XVA Metrics

In fact, in the current state of the market, it is only the CVA and, if any, the FVA (but the latter vanishes in our CCP setup without proprietary trading), which are effectively passed by banks to their clients through entry prices. The MVA and the KVA are mainly used by banks for collateral optimization purposes and detection of good/bad trade opportunities.

MVA compression of bilateral trade portfolios is considered in Kondratyev and Giorgidze (2017). In the context of centrally cleared derivatives, an application of the MVA as an optimization metric is mentioned in Sherif (2017). Banks do not only trade through CCPs to meet their clients' demand in terms of vanilla trades (newly issued ones have to be cleared by regulation). They also "clear their delta", i.e. hedge their books of exotics with clients through cleared vanilla trades. The point made in Sherif (2017) is then that, from an MVA optimization point of view, the most efficient way for banks to implement their hedges is by a combination of bilateral and intermediated cleared deals. Namely (including the proprietary trading account of the clearing member for the purpose of this discussion), proprietary cleared hedges are less efficient, in XVA terms, than otherwise equivalent combinations of bilateral trades and of intermediated (as opposed to proprietary) cleared trades. We illustrate this point by Figure 1, where the basic strategy on the bottom, implemented by the bank via its CCP proprietary account, is MVA inefficient with respect to the strategy on top. This is because, by IM regulatory specifications, a naked bilateral swaption position requires a higher level of initial margin than a covered bilateral swaption position. Moreover, the IMs attracted by cleared swaps are bound to be similar, whether the cleared swaps are traded with the bank itself through its proprietary account (as in the bottom) or with the client (as on top).

In view of this, banks should tend to migrate their (cleared) hedging portfolios from the basic strategy, implemented through their proprietary CCP account, to more MVA efficient delta clearing strategies implemented through their external client accounts. This is one of the reasons why we exclude the proprietary trading accounts from the main part of the paper (see also Sect. 2 and see Sect. 6.2 regarding the more general case including these accounts).

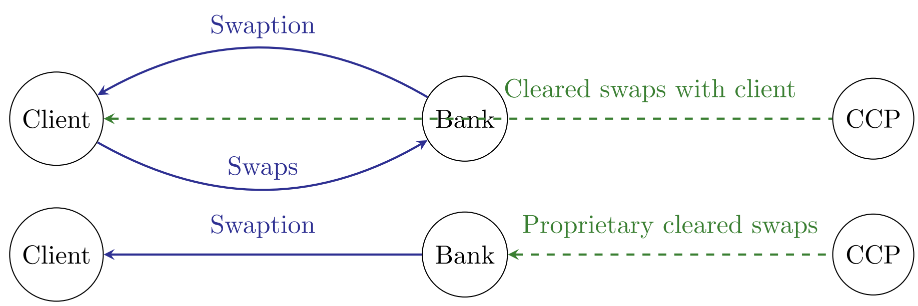

Figure 1: Clearing a swaption's delta: *(bottom)* Basic hedge implemented through the proprietary trading account of the clearing member bank, *(top)* MVA efficient hedge purely implemented through client trades. Bilateral transactions are displayed by solid blue arrows and centrally cleared transactions by dashed green ones. ∎

The KVA formula (8) can be used either in the "direct" mode, for an exogenous target hurdle rate h, or in the "implied" mode, for computing the implied hurdle rate h corresponding to the actual amount on the risk margin account of the clearing member. The core underlying idea is the establishment of a sustainable dividend remuneration policy to shareholders, as opposed to day one profit at every new deal. Cost of capital proxies have always been used to estimate return-on-equity. The KVA is a refinement, dynamic and fine-tuned for derivative portfolios, but the base return-on-equity concept itself is far older than even CVA.

3.6 Inefficiencies of Current CCP Designs

The XVA analysis of this section reveals several inefficiencies in the current design of CCPs and in the related regulatory framework:

- Potentially high cost of raising funding initial margins, if funded by unsecured borrowing (i.e. if $\lambda = \bar{\lambda}$ in (7));

- Ambiguous nature of the default fund contributions, between collateral and capital at risk (see Sect. 3.1), but tied up for sure in the default fund and not remunerated at a hurdle rate;
- Vexing modeling situation for the clearing members, which are not in a position of estimating their XVA metrics with accuracy (see end of Sect. 3.4);
- IM funding risk not capitalized under a Cover 2 specification of the default fund.

The following two sections are about possible ways out of these shortcomings.

4 CCP as a Centralized XVA Desk

In this section, we consider a CCP with extended mandates, which would also become a centralized XVA calculator and management center. The DFCs would become pure capital at risk of the CCPs, duly remunerated as such at a hurdle rate. In addition, the economic capital methodology of Albanese and Crépey (2019, Section 3.6) could be used, as an alternative to the current regulatory standards, for designing a sound specification of the default fund and/or of its allocation between the clearing members.

4.1 CCP Viewpoint

In what follows, the default fund is capital at risk of the clearing members, in the strict sense of a pure "survivor pay" resource. The margins, which are then the only "defaulter pay" resource in the system, should then be carefully devised to also account for the market liquidity impact of default resolutions (cf. Sect. 3.2).

The now pure capital at risk DFC^i terms therefore disappear from equation (1) and its dependencies, i.e. from ϵ in (4). Moreover, we propose a default fund also capitalizing for the risk related to the funding of the IM by the clearing members. The corresponding trading loss process of the CCP as a whole is written as

$$
\begin{aligned}
&L_0^{ccp} = 0 \text{ and, for } t \in (0,T],\\
&dL_t^{ccp} = \sum_{i=0}^{n} \Big((\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta^{-1}_{\tau_i^\delta}\beta_{\tau_i}(\mathrm{VM}^i_{\tau_i-} + \mathrm{IM}^i_{\tau_i-}) \Big)^+ \boldsymbol{\delta}_{\tau_i^\delta}(dt) + \sum_{i=0}^{n} J_t^i \lambda_t^i \mathrm{IM}_t^i dt \\
&\qquad + (d\mathrm{CVA}_t^{ccp} - r_t \mathrm{CVA}_t^{ccp})dt + (d\mathrm{MVA}_t^{ccp} - r_t \mathrm{MVA}_t^{ccp})dt.
\end{aligned} \tag{11}
$$

Here, for $0 \le t \le T$ (cf. Lemma 3.2 and recall that our CCP is default-free, hence does $\mathbb{Q}^*$ valuation),

$$
\begin{aligned}
\mathrm{CVA}_t^{ccp} &= \sum_{i=0}^{n} \mathbb{E}_t^* \mathbb{1}_{t<\tau_i^\delta<T} \beta_t^{-1} \Big(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{VM}^i_{\tau_i-} + \mathrm{IM}^i_{\tau_i-}) \Big)^+ \\
\mathrm{MVA}_t^{ccp} &= \sum_{i=0}^{n} \mathbb{E}_t^* \int_t^T \beta_t^{-1} \beta_s J_s^i \lambda_s^i \mathrm{IM}_s^i ds.
\end{aligned} \tag{12}
$$

Our alternative proposal for the sizing of the default fund will be based on the ensuing economic capital process of the CCP:

$$\mathrm{EC}_t^{ccp} = \mathbb{ES}_t^{*,a_{df}}\left(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s^{ccp}\right), \tag{13}$$

where a_{df} is a suitable quantile level. Our allocation of the default fund will be based on the corresponding member decremental

$$\Delta_i \mathrm{EC}^{ccp} = \mathrm{EC}^{ccp} - \mathrm{EC}^{ccp(-i)}. \tag{14}$$

Here $ccp(-i)$ refers to the CCP deprived from its i^{th} member, i.e. with the i^{th} member replaced by the risk-free buffer in all its CCP transactions.

4.2 Clearing Member Viewpoint

The trading loss process of the reference clearing member 0 corresponding to the above CCP design is given by the equation (7), but without the DFC^i terms in (4) in ϵ, and for the weight μ there defined by

$$\mu = \frac{J\Delta_0 \mathrm{EC}^{ccp}}{\sum_j J^j \Delta_j \mathrm{EC}^{ccp}} \tag{15}$$

(rather than (5)).

The KVA of the clearing member is sourced from end-users at deal inceptions and the ensuing amount on its risk margin account is loss-absorbing. Accordingly, the capital set at risk by the clearing member *shareholders* is only

$$(\mu \mathrm{EC}^{ccp} - \mathrm{KVA})^+ =: \mathrm{DFC}. \tag{16}$$

This results in a capital at risk contribution of the clearing member to the CCP, through its shareholders *and* end-users, of

$$(\mu \mathrm{EC}^{ccp} - \mathrm{KVA})^+ + \mathrm{KVA} = \max(\mu \mathrm{EC}^{ccp}, \mathrm{KVA}). \tag{17}$$

Remark 4.1 Our economic capital based DFCs are *not* meant as a variant of the regulatory capital that currently comes, along the lines of Basel Committee on Banking Supervision (2014), in addition to the default fund contribution of a member (and is typically negligible with respect to it, as it is meant to capitalize risk beyond it). See Armenti and Crépey (2017, Sections 4.5 and A.1) for more details on this regulatory capital of a clearing member bank. Under the alternative CCP design of this section, no regulatory capital would be required above the economic capital based DFCs, which would already be appropriately sized capital at risk of the clearing members. ∎

Our KVA of the clearing member is given by (8), but for DFC given by (16) there. As KVA then also enters the right-hand side in (8), we now deal with a KVA backward stochastic differential equation. However, the latter has a coefficient which is monotone in its KVA variable (assuming the process r bounded from below). By virtue of standard results (see e.g. Kruse and Popier (2016)), this KVA equation is therefore well-posed among square integrable solutions, assuming a square integrable process EC^{ccp}.

The total capital at risk of the CCP is the sum of the terms (17) related to each of the clearing members. This sum may be greater than EC^{ccp} (as the weights μ sum up to one), as it should (capital at risk of the CCP at least as large as its economic capital). The story then continues as in Sect. 3.3–3.4. The trading loss of the reference clearing member is still as in (7), but for ϵ there modified as described above. See Figure 2, which gives a graphical representation of the main involved cash flows (with $r = 0$, i.e. $\beta = 1$, to alleviate the picture).

The following result is a CCP adaptation of Albanese et al. (2019, Proposition 4.2) for bilateral trade portfolios. In order to rule out, for technical simplicity, jumps of our L or KVA processes at deal times, we assume a quasi-left continuous model filtration $\mathbb{G}$, as well as $\mathbb{G}$ predictable new deal (stopping) times. The former assumption excludes that martingales can jump at predictable times. It is satisfied in all practical models and, in particular, in all models with Lévy or Markov chain driven jumps. The latter is a reasonable assumption regarding the time at which a financial contract is concluded. It was actually already assumed regarding the (fixed) time 0 at which the CCP portfolio is supposed to have been set up in the first place.

Theorem 4.1 *Under the above dynamic and trade incremental cost-of-capital XVA strategy, the cumulative dividend stream of the reference clearing member shareholders is a $\mathbb{Q}^*$ submartingale on $\mathbb{R}_+$, with drift coefficient $h\mathrm{DFC}$ killed at τ.*

Proof. Between time 0 and before the next deal, the dividends to the shareholders of the reference clearing member (trading gains, risk premium payments, and OIS interest on the risk margin account) net to the process

$$-(dL_t + d\mathrm{KVA}_t - r_t\mathrm{KVA}_t dt), \tag{18}$$

stopped before time τ. Moreover, if the next deal time θ_1 is finite (and $< \tau$), the funds transfer policy defined by (10) allows the CCP to reset the reserve capital and risk margin accounts of each clearing member to their theoretical target values corresponding to the new CCP portfolio (including the new deal). This is done without contribution of the clearing members themselves: all the money required for these resets is (10), which is sourced from the clients of the deals. In addition, in a quasi-left continuous filtration, our L and KVA processes (the ones related to the initial portfolio, but also those, starting at time θ_1, related to the portfolio augmented with the new deal) cannot jump at the predictable time θ_1. Hence there are no clearing member shareholder dividends at θ_1 and the equality between the reference clearing member dividends and the process (18) stopped before τ holds until θ_1 *included*.

By the $\mathbb{Q}$ martingale property of L and the definition of the KVA, the process (18) is a $\mathbb{Q}$ submartingale with drift coefficient $h\mathrm{DFC}$. By Lemma 2.1, the process

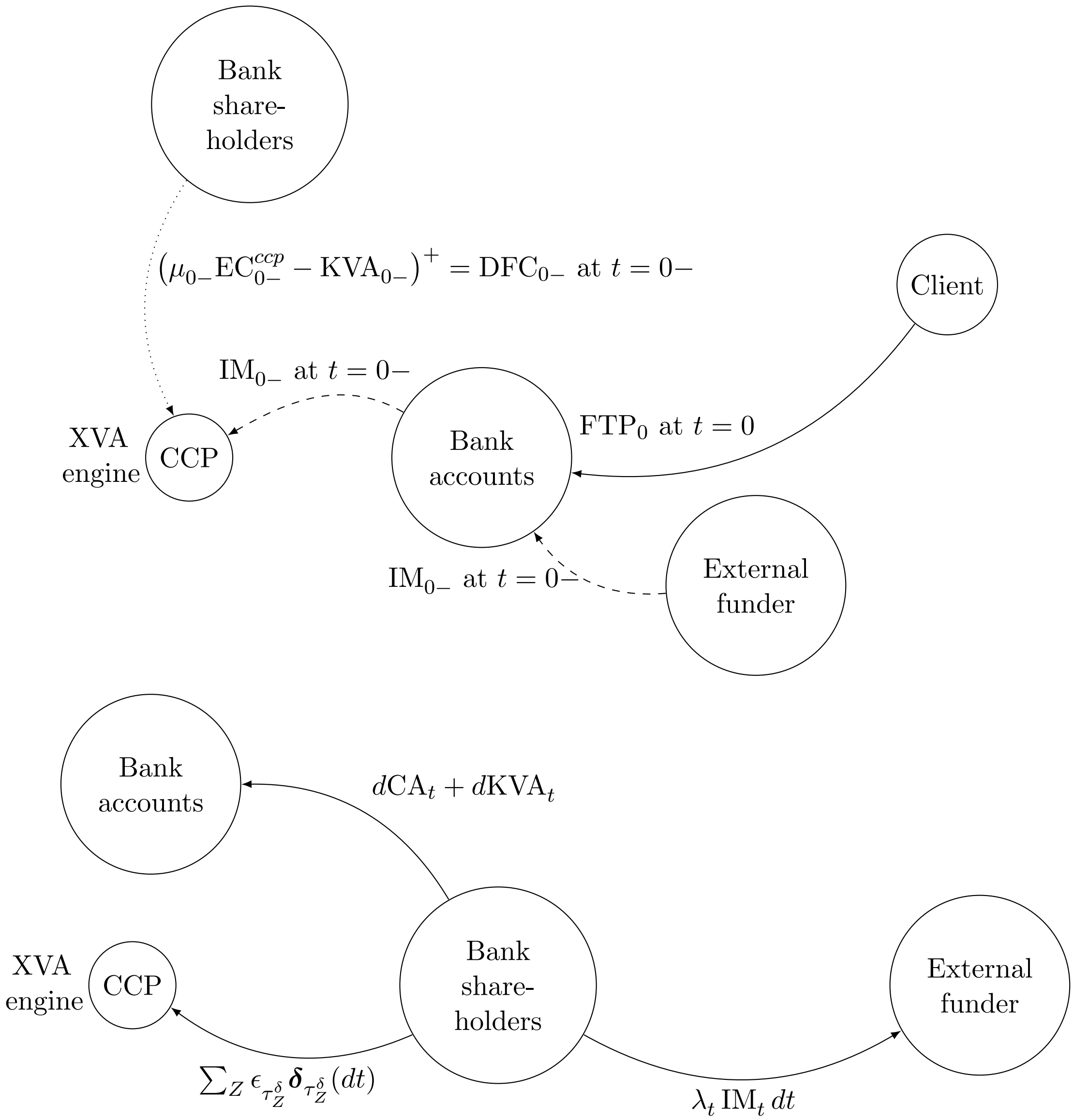

Figure 2: Cash-flows affecting the reference clearing member bank for $t < \tau$. *(Top)* At portfolio inception, with solid arrows for cash flows representing a transfer of property, dashed arrows for initial margin exchanges, and dotted arrows for default fund contribution movements. *(Bottom)* in $(t, t + dt]$ such that $0 < t < \theta_1 \wedge \tau$ (see the proof of Theorem 4.1), focusing on cash flows representing a transfer of property (and with $r = 0$) to alleviate the picture. ∎

(18) stopped before τ is therefore a $\mathbb{Q}^*$ submartingale, with drift coefficient hDFC killed before τ. In view of the above, so is therefore the clearing member shareholders' dividend process on $[0, \theta_1]$.

The reset at θ_1 puts us in a position to repeat the above reasoning relative to the time interval $[\theta_1, \theta_2]$, where θ_2 is the following deal time (i.e. on $[\theta_1, +\infty)$ if no next deal happens), and so on iteratively, so that the above submartingale property of the dividends holds on $\mathbb{R}_+$. ∎

We emphasize that the above proof, hence Theorem 4.1, is in fact independent of the specification of the default fund contributions, provided the latter are pure capital at risk of the clearing members. Again, as explained in Sect. 3.1, by current CCP designs, the default fund contributions are of an hybrid nature between "defaulter pay" collateral and "survivor pay" capital at risk. By contrast, as soon as the DFCs would be pure capital at risk of the clearing members (whether defined by (16), or still based on a Cover 2 or yet another specification), Theorem 4.1 means that clearing member shareholder earn dividends at a rate h on their capital at risk, which is the rationale of a cost-of-capital XVA approach with target hurdle rate h.

Remark 4.2 Theorem 4.1 relies on the assumption, made in the main body of this paper, that the reference clearing member only behaves as a market maker. Whenever a clearing member would behave as a market taker via a proprietary trading account, then the clearing member should incur a related FTP, just like an external client of the CCP. This gives another enlightening on the fact that the use of the proprietary account should be limited and, inasmuch as possible, dedicated to the purpose of hedging bilateral trades. We showed in Sect. 3.5 how this can be optimally done by shortcutting the proprietary account, repackaging cleared hedges (if implemented through the proprietary account initially) by means of bilateral and intermediated cleared deals. ∎

4.3 Discussion: Refined Risk Assessment vs. Model Risk

By comparison with the Cover 2 sizing rule, which purely relies on market risk, EC^{ccp} in (13) reflects a broader notion of risk of the CCP. Indeed, it corresponds to a risk measure of a one-year ahead trading loss-and-profit, as it results from the combination of the credit risk of the clearing members and of the market risk of their portfolios, including their IM funding risk.

Likewise, the allocation (15) of the default fund reflects the residual exposure of the CCP to the clearing members beyond their margins, like an STLOIM proportional allocation, but with a broader (hybrid market and credit) risk base.

Our DFCs are pure capital at risk (as opposed to collateral) of the clearing members, duly remunerated as such at a hurdle rate (return-on-equity) h.

As XVA computations are delegated to the CCP, the vexing modeling reported in Sect. 3.6 is solved. Our CCP would even be in a position to decide to which clearing member (or set of intermediating clearing members) a new deal (or package of deals) should be allocated, optimally in XVA terms, i.e. to the clearing member(s) for which the ensuing FTP^{ccp} (cf. (10)) is minimal.

More generally, the proposed setup and centralized XVA calculator at the CCP level, once in place, would offer a risk analysis environment that could be used for versatile purposes, including:

- XVA compression and collateral optimization;
- detection of XVA cross-selling opportunities;
- credit limits monitoring at the trade or counterparty level, with sensitivities to wrong way-risk, credit market and credit credit correlation (that are all missing with more rudimentary metrics such as potential future exposures);
- reverse stress testing in the context of CCAR exercises (comprehensive capital analysis and review US regulatory framework).

One should of course be vigilant about the model risk inherent to an economic capital based approach. In particular, the default risk of the clearing members is only known through their CDS spreads, and there is little joint default risk information. However, the model risk of our economic capital based approach can naturally be accounted for in a Bayesian variation of the KVA, where simulated paths of co-calibrated models are combined in the XVA simulations (cf. Hoeting, Madigan, Raftery, and Volinsky (1999)). The difference between such a KVA enhanced for model risk and a baseline KVA would naturally fall into the scope of an AVA (additional valuation adjustment), as envisioned by the European banking authority in order to account for various features including model risk.

By comparison, the Cover 2 sizing rule does of course not entail credit model risk. But it is not exactly robust in terms of credit risk either: it is only robust to "two defaults at the same time", cf. Murphy and Nahai-Williamson (2014, Section 6). Also note that a majority of the large CCPs do currently apply credit (ratings related) add-ons to, at least, initial margins (if not default fund contributions). Moreover, the Cover 2 approach does actually entail market model risk, and an arguably even less controllable one than the one embedded in an economic capital based approach. The corresponding model risk is hidden in the ubiquitous notion of "extreme but plausible" scenarios (see Sect. 3.1).

Obviously, an economic capital based approach is only conceivable in the case of a CCP with solid modeling skills, as a regulator could have all freedom to assess. But, again, some key points of our proposed approach in this section, namely the suggestion that a CCP be itself involved into the XVA computations, or that DFCs be remunerated at some hurdle rate (of the order a few percents, not basis points, above OIS), do not necessarily require to switch to an economic capital based default fund. They are compatible with any methodology of choice (Cover 2 or other) for computing the latter. The economic capital based approach could also be restricted to the computation of the allocation, between the clearing members, of a Cover 2 default fund. Finally, even if an economic capital based approach is not used for sizing the DFCs in production, we believe that it is interesting both theoretically and as a risk investigation tool for a CCP and its clearing members (cf. our case study in Sect. 7).

5 Margin Lending

It remains to address the first inefficiency mentioned in Sect. 3.6, namely the potentially high cost of initial margin if funded by unsecured borrowing.

The unsecured borrowing spread process of the reference clearing member bank in Sect. 3 can be proxied by $\bar{\lambda} = \gamma(1-R)$, where γ is its risk-neutral default intensity and R is its recovery rate. The best proxy for a term structure of $\bar{\lambda}$ is the CDS curve of the bank. As bank hurdle rates (e.g. 10%) are typically one order of magnitude greater than their CDS spreads, unsecured borrowing of IM is a cost efficient strategy if compared to using equity capital (i.e. own assets) for this purpose.

Accordingly, as of today, a common IM raising policy is unsecured borrowing. The external funder loses $(1-R)\beta_{\tau^\delta}^{-1}\beta_\tau \mathrm{IM}_{\tau-}$ at τ^δ, which the external funder charges $\bar{\lambda}_s \mathrm{IM}_s ds$ to the bank until its default (see Figures 2 bottom and 3 top). This is fair to the external funder (assumed default-free) in view of the identity

$$\mathbb{E}_t^*\Big[\beta_{\tau^\delta}\mathbb{1}_{t<\tau<T}(1-R)\beta_{\tau^\delta}^{-1}\beta_\tau \mathrm{IM}_{\tau-}\Big] = \mathbb{E}_t^*\left(\int_t^{\tau\wedge T}\beta_s\bar{\lambda}_s\mathrm{IM}_s ds\right),\ 0\le t\le T. \tag{19}$$

However (see the bottom panel of Figure 3), we can consider an alternative initial margin lending scheme, whereby the IM is provided to the CCP on behalf of the bank by a so called specialist lender, in exchange of some service fee. Under the terms of a legal agreement concluded between the CCP and the specialist lender, in case of default of the clearing member, the specialist lender would receive back from the CCP the portion of IM unused to cover losses. Note that such, as margin lending is implemented off the balance sheet of the clearing member bank, it is not a violation of pari passu rules. It is just a form of collateralized lending: see Sect. 5.1 for more corporate details and implications.

We assume that the specialist lender funds are kept at the segregated account for initial margin. In case of default, this account is depleted by the amount $(G_{\tau^\delta}^+ \wedge \beta_{\tau^\delta}^{-1}\beta_\tau \mathrm{IM}_{\tau-})$, where the time-$t$ gap G_t is given as

$$G_t = \mathrm{MtM}_t + \Delta_t - \beta_t^{-1}\beta_{t-\delta}\mathrm{VM}_{(t-\delta)-}, \tag{20}$$

and the lender receives a claim against the bank estate. Hence, the exposure of the specialist lender to the default of the clearing member is

$$(1-R)(G_{\tau^\delta}^+ \wedge \beta_{\tau^\delta}^{-1}\beta_\tau \mathrm{IM}_{\tau-}). \tag{21}$$

Let ξ be a $\mathbb{G}$ predictable process, which exists by Corollary 3.23 2) in He, Wang, and Yan (1992), such that

$$\beta_\tau\xi_\tau = \mathbb{E}_{\tau-}^*\big[(\beta_{\tau^\delta}G_{\tau^\delta}^+ \wedge \beta_\tau \mathrm{IM}_{\tau-})\big] \le \beta_\tau \mathrm{IM}_{\tau-}. \tag{22}$$

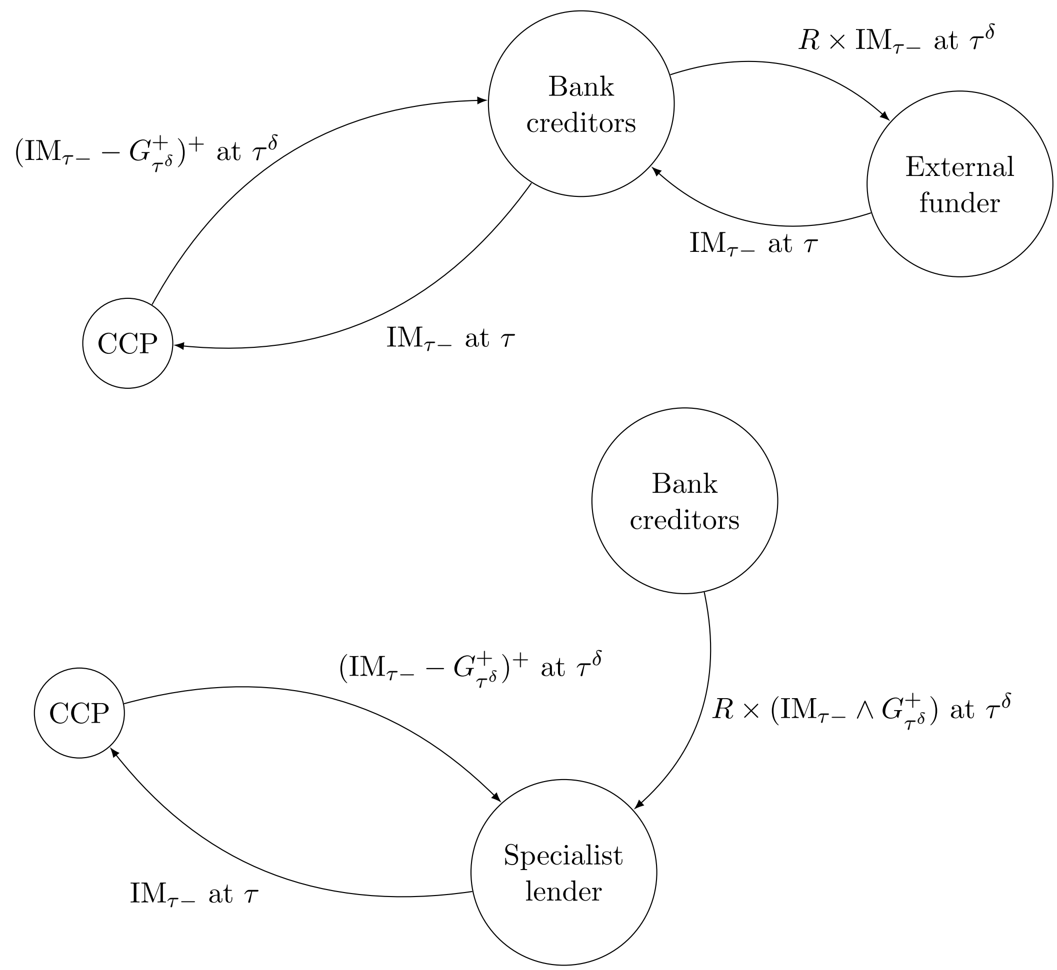

Figure 3: Reference clearing member bank own-default related funding cash-flows, with $r = 0$ to alleviate the picture. The specialist lender on the bottom panel is lending the same IM amount to the CCP (on behalf of the bank) than the external funder is lending to the bank (who is then lending it to the CCP) on top. But, in case the bank defaults, the specialist lender receives back from the CCP the portion of IM unused to cover losses. Hence it is reimbursed at a much higher effective recovery rate than the nominal recovery rate R embedded in the bank credit spread.

We assume that the clearing member, as long as it is non-default, pays continuous-time service fees $\bar{\lambda}_s \xi_s ds$ to the specialist lender.

Summing up, the specialist lender loses $(1-R)(\mathrm{IM}_{\tau-} \wedge G^+_{\tau^\delta})$ at τ^δ, which the specialist lender charges $\bar{\lambda}_s \xi_s ds$ to the bank until its default. Such arrangement can be deemed fair to the specialist lender (assumed risk-free), in view of the identities

$$\begin{aligned}
&\mathbb{E}^*_t\Big[\beta_{\tau^\delta}\mathbb{1}_{t<\tau<T}(1-R)\big(G^+_{\tau^\delta}\wedge\beta^{-1}_{\tau^\delta}\beta_\tau\mathrm{IM}_{\tau-}\big)\Big]\\
&\qquad=\mathbb{E}^*_t\Big[\mathbb{1}_{t<\tau<T}(1-R)\mathbb{E}^*_{\tau-}\big[\beta_{\tau^\delta}\big(G^+_{\tau^\delta}\wedge\beta^{-1}_{\tau^\delta}\beta_\tau\mathrm{IM}_{\tau-}\big)\big]\Big]\\
&\qquad=\mathbb{E}^*_t\Big[\mathbb{1}_{t<\tau<T}\beta_\tau(1-R)\xi_\tau\Big]=\mathbb{E}^*_t\left(\int_t^{\tau\wedge T}\beta_s\bar\lambda_s\xi_s ds\right),\ 0\le t\le T.
\end{aligned}\tag{23}$$

Theorem 5.1 *Assuming risky funding fairly priced by the market, the respective risky funding spreads $\lambda=\lambda^{ub}$ and $\lambda=\lambda^{sl}$ (cf. Sect. 3.3) of the bank corresponding to unsecured borrowing and margin lending are $\lambda^{ub}=\bar\lambda$ and λ^{sl} such that*

$$\frac{\lambda^{sl}}{\bar\lambda}=\frac{\xi}{\mathrm{IM}_-}\le 1.\tag{24}$$

The associated MVA = MVA^{ub} *and* MVA = MVA^{sl} *are such that, for $t<\tau$,*

$$\mathrm{MVA}^{ub}_t=\mathbb{E}_t\left(\int_t^T\beta_t^{-1}\beta_s\bar\lambda_s\mathrm{IM}_s ds\right)\le\mathbb{E}_t\int_t^T\beta_t^{-1}\beta_s\bar\lambda_s\xi_s ds=\mathrm{MVA}^{sl}_t.\tag{25}$$

Proof. The identities (19) and (23), compared with our generic IM instantaneous risky funding cost specification $\lambda_s\mathrm{IM}_s ds$ (see before Definition 3.1), show that $\lambda^{ub}=\bar\lambda$ and $\lambda^{sl}=\frac{\xi\bar\lambda}{\mathrm{IM}_-}$, respectively. Our generic MVA formula in (6) then yields the identities in (25). The inequality in (24), which immediately implies the one in (25), stems from the one in (22). ∎

5.1 Discussion: Credit and Liquidity

We emphasize that the above result is independent of the specification of the IM process, possibly inclusive of any add-ons such as the ones mentioned in Sect. 3.2, as long as the IM (add-ons included) is provided by the specialist lender.

Moreover, under a basic specification where $\beta_s\mathrm{IM}_s$ is set as a high quantile (time-t conditional value-at-risk) of $\beta_{s^\delta}G_{s^\delta}$ (cf. (29) below, assuming there for simplicity continuous-time variation margining $\mathrm{VM}_t=\mathrm{MtM}_t$ before time τ in (20)), then the blending ratio in (24) is typically much less than 1, i.e. $\lambda^{sl}\ll\bar\lambda$. Hence MVA^{sl} should be *significantly less* than MVA^{ub} . If DFCs are also usable for own default settlement as per current CCP designs (of course, this point becomes irrelevant under our pure capital DFC proposal of Sect. 4), subordinating own DFC to IM would result in less IM consumption upon defaults, with $(G_{\tau^\delta}-\mathrm{DFC}_{\tau-})^+$ instead of $G^+_{\tau^\delta}$ in (21), hence even more efficient IM lending schemes

From a "wrong way risk" perspective, one may be worried that the margin lending service fee, $\bar\lambda_s\xi_s ds$ in the above mathematical setup, increases with the default risk of the clearing member bank (represented by $\bar\lambda_s$). But this is also the case of the instantaneous cost, $\bar\lambda_s\mathrm{IM}_s ds$, for unsecured borrowing of the IM. In fact, the ratio (24) between the two costs does not depend on $\bar\lambda$.

Margin lending practicalities are an active investor (such as private equity fund) business, whereas the available liquidity is mostly with passive investors (insurance, pension funds,...). Hence, the implementation of margin lending naturally calls for a two-layered structure, whereby an active investor bridges to a passive one. Now, the above developments (whether they regard margin lending or unsecured borrowing) are only on the credit side of the problem, with short-term funding assumed continuously rolled over in time. As always with credit, there is another, liquidity side to it. This is the fact that the lender (the so called external lender in the case of unsecured borrowing and specialist lender in the case of margin lending) may want to cease to roll-over its loan, not because of the credit risk of the borrower, but just because of liquidity squeeze in the market: the lender may be short of cash (or liquid assets), or want to keep the latter for other (own) purposes. It may then be argued that the liquidity issue is more stringent for margin lending, with its two-layered structuring, than for unsecured borrowing. This is also why margin lending is more difficult to implement for VM than for IM (cf. before Sect. 1.1). In order for an IM margin lending business to obtain the blessing of a regulator, it would be better to address this liquidity issue in the legal structure of the setup. This could for instance take the form of an option for the clearing member bank to force one roll-over by the specialist lender (once in the life of the contractual relation, say, and in exchange of a then higher interest rate). Such a “liquidity option” would have to be priced into the structure. Of course, unsecured borrowing is not exempt of liquidity issues either. A detailed comparison of unsecured borrowing and margin lending also accounting for these liquidity issues could be an interesting topic of further research.

6 Extensions: Defaultable End-Users and Hybrid Portfolios

6.1 Defaultable End-Users

To deal with the realistic case of defaultable end-users in our setup, the visibility scope of the CCP needs be extended in order to encompass the latter (beyond the clearing members). Risky end-users then just means additional default times (the ones of the external clients) and corresponding new terms in the CCP equations (11) and (12), as well as in the clearing member CVA (6) (without the DFC^i terms in (4) in ϵ, and with μ there preferably defined by (15)). The main difference between an external client and a clearing member is that the former (may be entitled to do proprietary trading, cf. Sect. 6.2, and) do not contribute to the default fund. Note that, in our proposed setup, the latter does not induce any unfairness, as default fund contributions are remunerated at a hurdle rate.

By contrast, leaving external clients outside the scope of the CCP (as they are under current CCP designs) implies that an external client default is only perceived by the CCP as “new deals”, corresponding to the unwinding by the clearing members of their positions with the defaulter. Such new deals should be charged at their FTP by the CCP to the clearing members. The clearing members should then account for such future FTPs at each client default node of their bilateral XVA simulations (cf. Sect. 6.2). But to compute the latter, the CCP should run nested XVA simulations conditional on every future default of a clearing member. Such computations are clearly unworkable, which reflects an inefficient split between the mandates of the CCP and the ones of the clearing members.

6.2 Centrally Cleared and Bilateral Portfolios

In the case of bilateral transactions, the XVA desks of the bank filter out counterparty risk and its capital and funding implications from client trades. Thanks to their entremise, the other trading desks of the bank can focus on the market risk of their business lines, as if there was no counterparty risk (see Albanese and Crépey (2019, Section 3.2)).

In the case of centrally cleared transactions, the role of the XVA desks is played by the CCP. Intermediated cleared trades, with end-users, are back-to-back hedged in terms of market risk. Proprietary cleared trades can be used for hedging bilateral trades of the bank. However, as illustrated in Sect. 3.5, such hedges tend to be less efficient, in XVA terms, than otherwise equivalent combinations of bilateral trades and of intermediated (as opposed to proprietary) cleared trades. Hence one may argue that proprietary cleared trading should tend to vanish "at equilibrium".

To address the realistic case of a bank involved into both bilateral and centrally cleared derivative transactions, including cleared proprietary trading if any, one applies the analysis of this paper for each CCP in which the bank is involved as a clearing member. The VM funding needs (positive or negative) required by the proprietary trading of the bank in different CCPs, merged with the one stemming from its bilateral trading, should then be plugged into a global FVA computation at the overall bank level. This FVA computation, as well as the XVA analysis of the bilateral trading of the bank, can be performed along the lines already developed in Albanese and Crépey (2019).

Hence, the combination of Albanese and Crépey (2019) and of the present paper allows dealing with the XVA analysis of a bank involved into an arbitrary combination of bilateral and centrally cleared portfolios. More precisely, the XVA computations of a bank can be split into the following:

- central clearing CVA, MVA, and KVA computations that can be performed at the level of each CCP into which the bank is involved as a clearing member. For more efficiency in XVA terms, these computations should better be done by each CCP itself (rather than left to the members that do not have the full required information). Each corresponding KVA is the cost of the capital set at risk by the bank in the default fund. The default fund contributions could (but do not need to) be computed by an economic capital based approach, accounting not only for the risk of counterparty default losses, but also for the liquidity funding risk related to the raising of initial margin;

- bilateral CVA, FVA, MVA, and KVA computations, where the CVA and MVA metrics can be computed at the level of each bilateral netting set, whereas the FVA and the KVA need be computed at the level of the overall bilateral trading book of the bank (also accounting for the VM data stemming from cleared proprietary trading, if any).

7 Case Study

Back to the core CCP setup of Sect. 2 through 5, we proceed with a numerical case study in the CCP toy model described in Sect. A. This model has a single Black–Scholes market risk factor and all the credits are independent from the market. Hence semi-explicit formulas are available for all the useful quantities in the paper. It is only (a term structure of) the economic capital of the CCP that needs be obtained by simulation as explained in Sect. 7.2.

The actual number of members in CCP services varies from four or five in starting services to several hundreds on certain asset classes. However, most CCP services are driven by no more than a dozen of major players, with typically two or three prominent ones (see e.g. Armenti, Crépey, Drapeau, and Papapantoleon (2018, Sections 6.1 and C)). Accordingly, in our numerics, we consider a pool of nine members only (to alleviate the computational load), but well diversified in terms of market and credit risk, which are the two main features of interest for our purposes.

We use $m = 10^5$ simulated paths of the underlying Black–Scholes rate S and clearing member default scenarios. ***All the reported numbers are in basis points. The nominal "Nom" of the swap that the clearing members are trading is fixed so that each leg equals*** $1 = 10^4$ ***basis points at time 0***.

In our numerics, we stay with the basic formulation of an initial margin given by a (risk-neutral and forward looking) value-at-risk (cf. Sect. 3.2), but we will play with the quantile level a_{im} used for setting the IM in order to emulate more or less conservative initial margining schemes. Unless stated otherwise, we assume the IM funded by unsecured borrowing, $a_{im} = 85\%$ as the quantile level of the value-at-risk used for setting their IM (cf. (29)), and $a_{df} = 97.5\%$ as the level of our expected shortfall specification (13) of the economic capital of the CCP. The resulting XVA numbers should be considered not so much in absolute terms than in relative terms between the clearing members and in terms of sensitivities with respect to the market and credit risks of the latter.

7.1 Margin Valuation Adjustments

In the context of our toy model, with deterministic (pre-)default intensities of the clearing members, the funding spread blending ratio of margin lending vs. unsecured borrowing in (24) is given by the constants (40)-(41). These constants only depend on the clearing member i through its direction, long or short, in the underlying swap (cf. Table 3 in Sect. A.2). They are displayed in Figure 4 as a function of the IM quantile level a_{im} in (32). In line with the theoretical analysis of Sect. 5, these blending ratios are significantly smaller than one, all the more so for higher quantile levels.

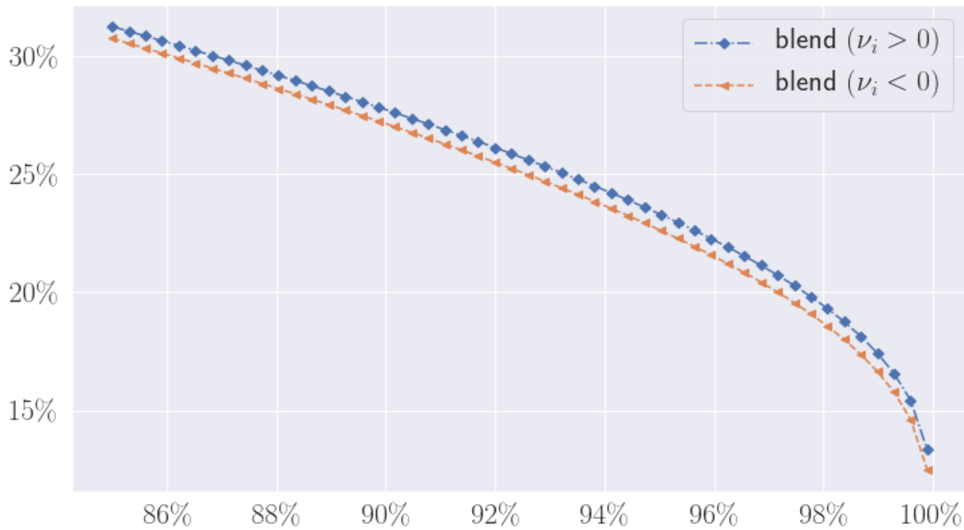

Figure 4: Funding spread blending ratio (24) of margin lending w.r.t. unsecured borrowing as a function of the IM quantile level a_{im}, for clearing members short ($\nu_i < 0$) vs. long ($\nu_i > 0$) in the swap. ∎

Figure 5 shows the ensuing time-0 MVAs of the nine clearing members for unsecured borrowing (blue) vs. margin lending (orange) of the IM, for $a_{im} = 85\%$ (bottom), 95% (middle), and 99.5% (top). As predicted by the theory at the end of Sect. 5, the margin lending MVAs of the clearing members are several times cheaper than their unsecured borrowing MVAs.

7.2 Economic Capital of the CCP

For simplicity we focus on EC^{ccp} as a proxy of the capital at risk of the CCP (accounting for the KVA components, the actual capital at risk is the sum of the terms analogous to (17) regarding each of the clearing members, which may be greater than EC^{ccp}).

The blue curves in Figure 6 show the resulting default fund term structures in the sense of the deterministic *functions* $\mathbb{ES}^{*,a_{df}}\left(\int_t^{t+1}\beta_t^{-1}\beta_s dL_s\right)$ (as a mean-field proxy to the *process* in (13)), for $a_{df} = 95\%, 97.5\%$, and 99% (bottom to top, while a_{im} is fixed to 85%). The other curves represent the analog results focusing on the terms in the first line in (11) (curves EC^{raw}), on the second line (curves EC^{comp}, where comp refers to "compensator"), on the default and CVA terms (curves EC^{def}), and on the IM and MVA terms (curves EC^{im}). Left panels are for unsecured borrowing of IM and right panels are for margin lending.

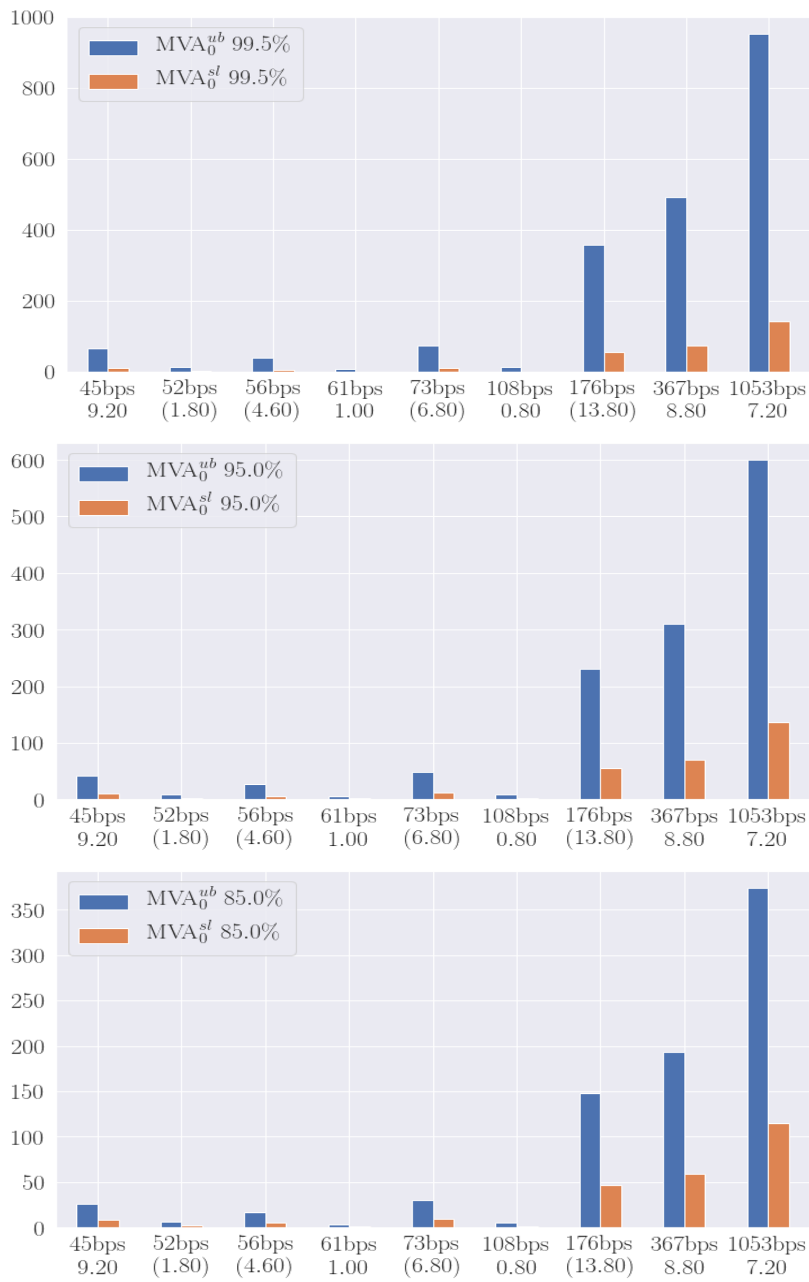

Figure 5: MVAs of the nine clearing members for unsecured borrowing *(blue)* vs. margin lending *(orange)* IM raising policies, for $a_{im} = 85\%$ *(bottom)*, 95% *(middle)*, and 99.5% *(top)*. ∎

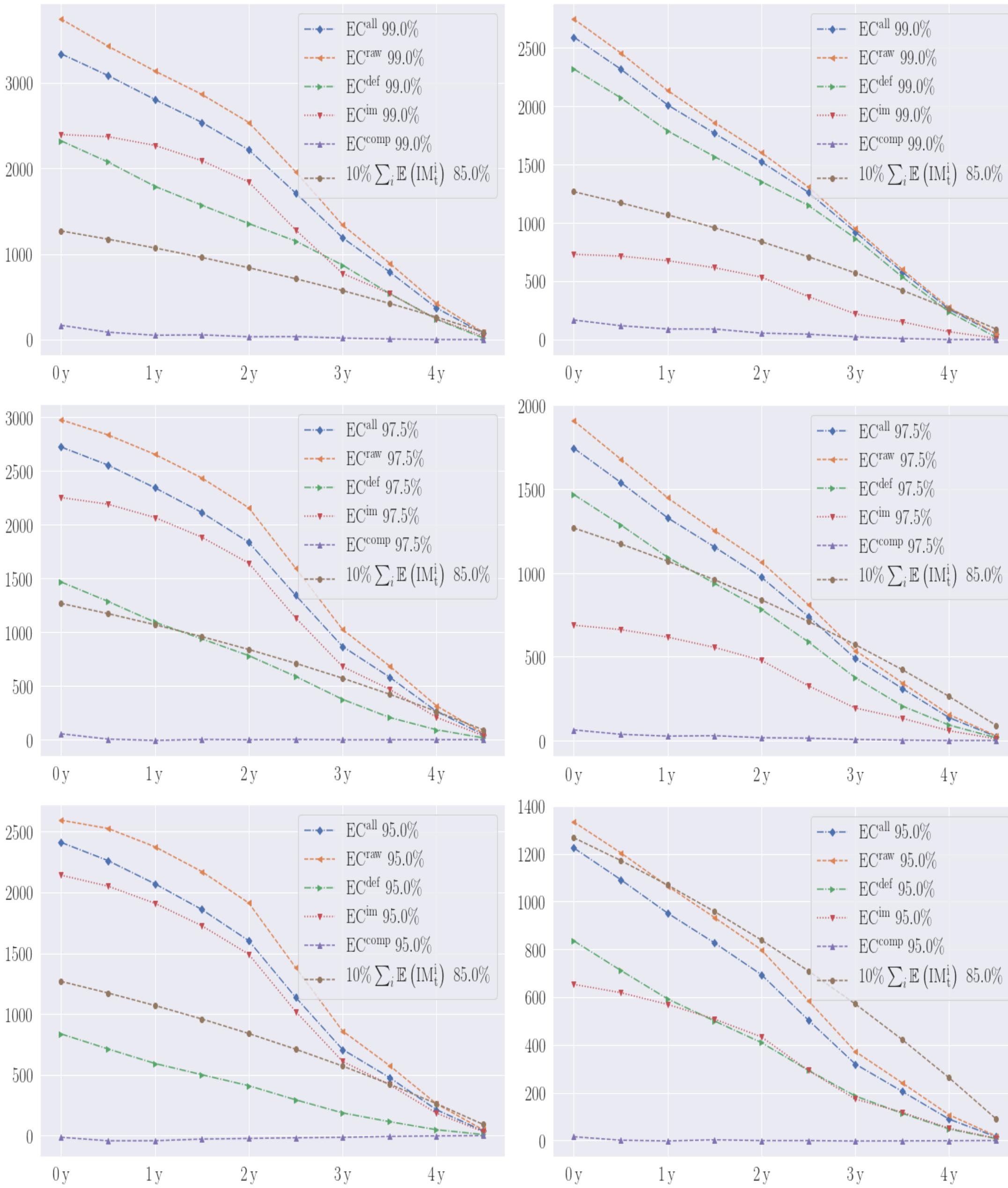

Figure 6: *Blue curves*: Economic capital based default fund of the CCP, as a function of time, for $a_{df} = 95\%, 97.5\%$, and 99% (*bottom to top*), while $a_{im} = 85\%$. *Other curves*: Analog results only considering the terms in the first line in (11) (curves EC^{raw}), the terms in the second line (curves EC^{comp}), the default and CVA terms (curves EC^{def}), and the IM and MVA terms (curves EC^{im}). The $10\% \times$IM term structure is also shown on all graphs as a proxy of a Cover 2 specification of the default fund. *Left panels*: Unsecured borrowing of IM. *Right panels*: Margin lending of IM. ∎

The broadly decreasing feature of all curves reflects the run-off feature of portfolio-wide XVA computations (cf. Sect. 3.4). The comparison between the different curves in each panel shows that the main contributors to the risk of the CCP are the volatility swings of the default and IM losses themselves, rather than those of the corresponding CVA and MVA liabilities. This was expected given, in particular, the deterministic intensity model that we use for the default times. Extreme swings of CVA^{ccp} could only arise in more structural "distance to default" credit models, where defaults are announced by volatile swings of CDS spreads (but calibration of default dependence is then a much more challenging issue than with our common shock model). Moreover, in the case of unsecured borrowing, the IM expenses create even more risk than the default losses, whereas the opposite prevails in the case of margin lending. This numerical observation is an illustration of the transfer of counterparty risk into liquidity (funding) risk triggered by extensive collateralization (cf. Cont (2017)). It clearly challenges the current regulatory Cover 2 sizing rule of the default fund, which is purely focused on default losses, ignoring the impact of the IM expenses.

On each panel we also display the term structure of 10%×the IM aggregated across all clearing members, as an indication of what a Cover 2 default fund could be (cf. Sect. 3.2) by comparison with our economic capital based proposal. We stress again that own DFCs are not usable to cope with own default under our alternative CCP design proposal, so that a default fund larger than Cover 2 (but remunerated at a hurdle rate) is reasonable (unless the IMs would themselves be higher).

The vertical comparison between the different panels of Figure 6 allows assessing the sensitivity of the results to more or less conservative quantile levels for the expected shortfall that is used in the economic capital computations.

7.3 Allocation of the Default Fund

Figure 7 shows the time-0 default fund allocation weights corresponding to member IM versus member decremental EC^{ccp} proportional rules, respectively represented by blue and orange bars. In the lower panel the clearing members are ordered along the x axis by increasing position $|\nu_i|$, whereas in the upper panel they are ordered by increasing credit spread Σ_i (cf. Table 3). In our CCP toy model where all portfolios are driven by a single Black-Scholes underlying (see Sect. A.1), the initial margins, hence the blue bars in Figure 7, are simply proportional to the size $|\nu_i|$ (or nominal Nom_i) of the member positions. By contrast, the member decremental EC^{ccp} allocations (orange bars) also take the credit risk of the members into account. They also encompass the risk created by the ensuing IM expenses, which, as revealed by Figure 6, happens to be an important, if not the main, contributor to EC^{ccp}.

Figure 8 shows term structures of IM based versus member decremental EC^{ccp} based DFC allocation weights, for each of the clearing members. We clearly see the impact of market but also credit risk on the EC^{ccp} based term structures, whereas the IM based allocations are constant through time. The latter is due to the fact that, in the setup of our toy model, IMs are proportional to the nominal positions of the clearing member.

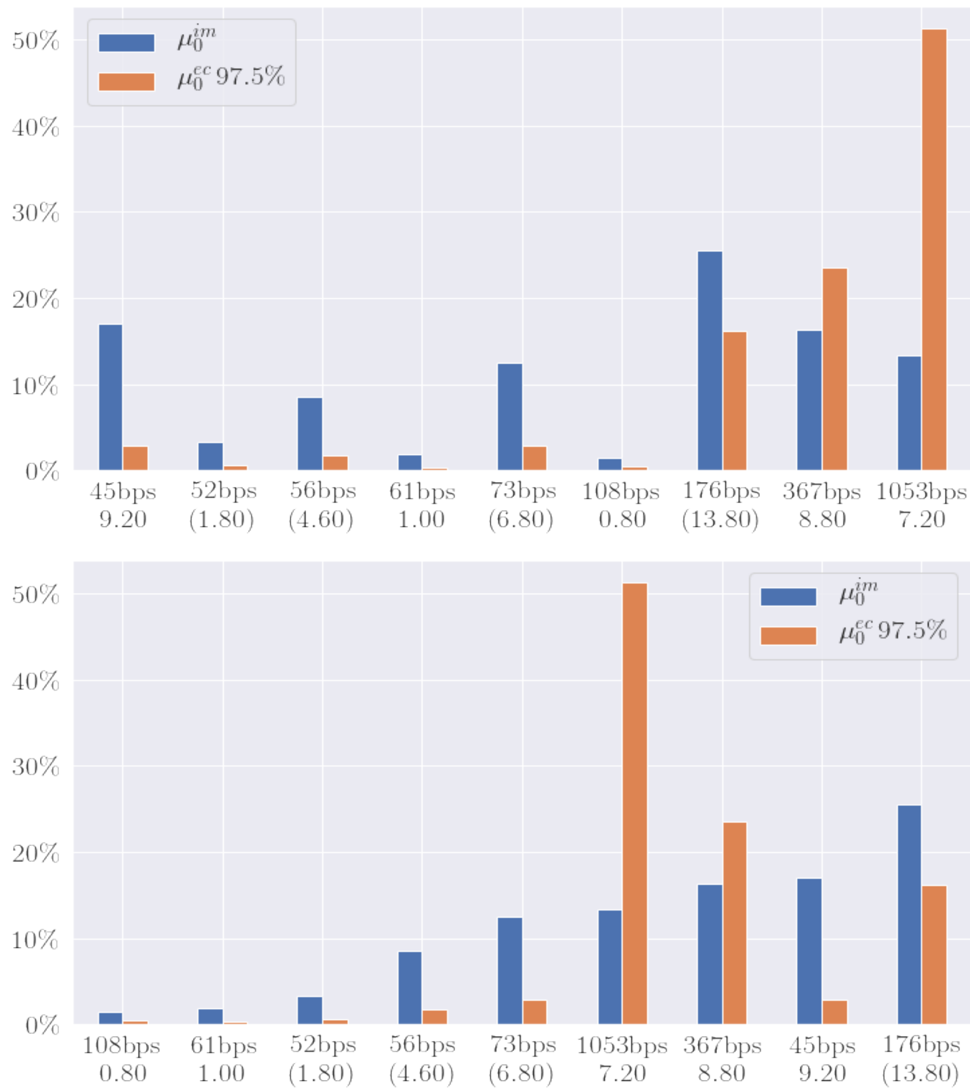

Figure 7: Time-0 default fund allocation based on member initial margin and member decremental EC^{ccp}. *Bottom*: Members ordered by increasing position $|\nu_i|$. *Top*: Members ordered by increasing credit spread Σ_i. ∎

Figure 8: Default fund allocation weights term structures based on initial margins *(in orange)* versus member decremental EC^{ccp} *(in blue)*, for each member, ordered from left to right and top to bottom per increasing credit spread, as a function of time $t = 0, \ldots, 4.5$. ∎

7.4 Comparative XVA Metrics Under Our Alternative CCP Design Proposal

Assuming the alternative CCP design of Sect. 4, Table 2 displays the three time-0

	ub	sl
$\sum_{i=0}^{n} J_0^i \mathrm{CVA}_0^i$	102.23	102.23
$\sum_{i=0}^{n} J_0^i \mathrm{MVA}_0^i$	804.25	248.28
$\sum_{i=0}^{n} J_0^i \mathrm{KVA}_0^i$	610.90	353.87
FTP_0^{ccp}	1517.38	704.38

Table 2: Economic capital based CVA_0^{ccp}, MVA_0^{ccp} and KVA_0^{ccp}, under unsecured borrowing *(left)* vs. margin lending *(right)* IM raising policies. ∎

XVA metrics aggregated over all clearing members, under our alternative CCP design proposal, as well as the corresponding portfolio-wide FTP^{ccp} (cf. (10)), in the cases of unsecured borrowing vs. margin lending IM raising policies. In both cases, the MVA and the KVA are the dominant metrics (the CVA, which does not depend on the IM raising strategy, is always smaller). The MVA is greater than the KVA in the case of unsecured borrowing of IM, and vice versa in the case of margin lending. The portfolio-wide FTP^{ccp} is about halved by switching from unsecured borrowing to margin lending. Figure 9 displays the corresponding member decremental metrics ($\mathrm{XVA}^{ccp} - \mathrm{XVA}^{ccp(-i)}$) (see after (14)). These differences are interpreted as costs triggered by member i to the CCP (whereas the CVA in (6) is a cost triggered to the reference clearing member by the CCP). In particular, ($\mathrm{CVA}^{ccp} - \mathrm{CVA}^{ccp(-i)}$) and ($\mathrm{MVA}^{ccp} - \mathrm{MVA}^{ccp(-i)}$) correspond to the summands in (35).

A CCP Toy Model

When a systematically important financial institution defaults, the impact on interest rates and foreign exchange rates is bound to be major. In the XVA analysis of centrally cleared derivatives, a model of joint defaults and a granular simulation of the latter is necessary if one wants to be able to account for the corresponding "hard wrong-way risk" issue. A credit portfolio model with particularly good calibration and defaults simulation properties is the common shock or dynamic Marshall-Olkin copula model of Crépey, Bielecki, and Brigo (2014, Chapt. 8–10) and Crépey and Song (2016) (see also Elouerkhaoui (2007, 2017)).

In this section we describe the corresponding CCP simulation setup, which is used in the numerics of Sect. 7. In particular, CVA^{ccp} and MVA^{ccp} are analytic in this model, which avoids the numerical burden of nested or regression Monte Carlo schemes that are required otherwise for simulating the trading loss processes involved in the economic capital computations.

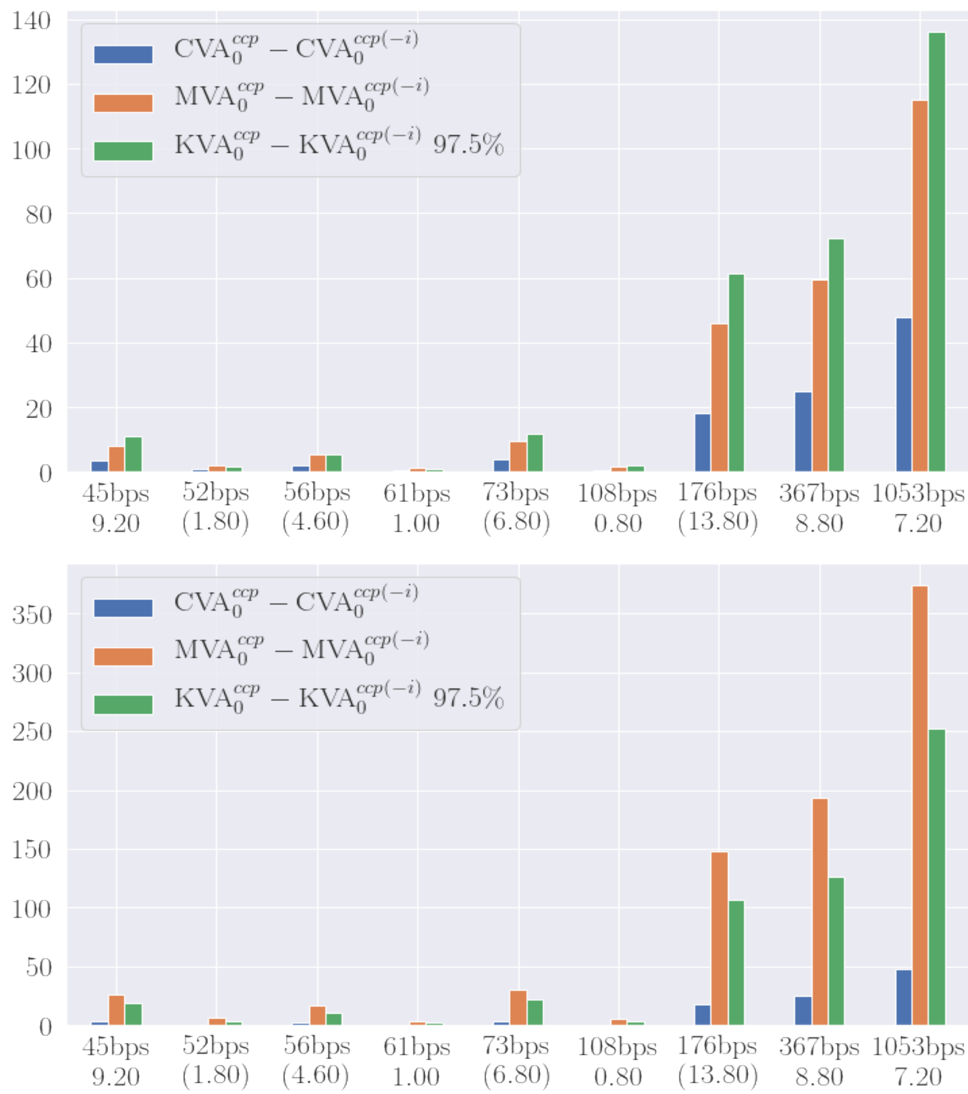

Figure 9: Member decremental CVA^{ccp}, MVA^{ccp}, and KVA^{ccp} of the nine clearing members, for unsecured borrowing *(bottom)* vs. margin lending *(top)* IM raising policies. ∎

A.1 Market Model

As common asset driving all our clearing member portfolios, we consider a stylized swap with strike rate $\bar{S}$ and maturity $\bar{T}$ on an underlying (FX or interest) rate process S. At discrete time points T_l such that $0 < T_1 < T_2 < ... < T_d = T$, the swap pays an amount $h_l(S_{T_{l-1}} - \bar{S})$, where $h_l = T_l - T_{l-1}$. The underlying rate process S is supposed to follow a standard Black-Scholes dynamics with risk-neutral drift κ and volatility σ, so that the process $\widehat{S}_t = e^{-\kappa t} S_t$ is a Black martingale with volatility σ. For $t \in [T_0 = 0, T_d = \bar{T}]$, we denote by l the index such that $T_{l_t - 1} \le t < T_{l_t}$. The mark-to-market of a long (floating receiving) position in this swap is given by

$$\begin{aligned} \mathrm{MtM}_t^\star &= \mathrm{Nom} \times \mathbb{E}_t^*\big[\beta_t^{-1}\beta_{T_{l_t}} h_{l_t}(S_{T_{l_t-1}} - \bar{S}) + \sum_{l=l_t+1}^{d} \beta_t^{-1}\beta_{T_l} h_l(S_{T_{l-1}} - \bar{S})\big] \\ &= \mathrm{Nom} \times \Big(\beta_t^{-1}\beta_{T_{l_t}} h_{l_t}(S_{T_{l_t-1}} - \bar{S}) + \beta_t^{-1} \sum_{l=l_t+1}^{d} \beta_{T_l} h_l\big(e^{\kappa T_{l-1}}\widehat{S}_t - \bar{S}\big)\Big), \quad (26) \end{aligned}$$

by the martingale property of the Black process $\widehat{S}$.

The following numerical parameters are used:

$$r = 2\%,\ S_0 = 100,\ \kappa = 12\%,\ \sigma = 20\%,\ h_l = 3 \text{ months},\ \bar{T} = 5 \text{ years}.$$

The nominal (Nom) of the swap is set so that each leg has a time-0 mark-to-market of one (i.e. 10^4 basis points). Figure 10 shows the resulting mark-to-market process

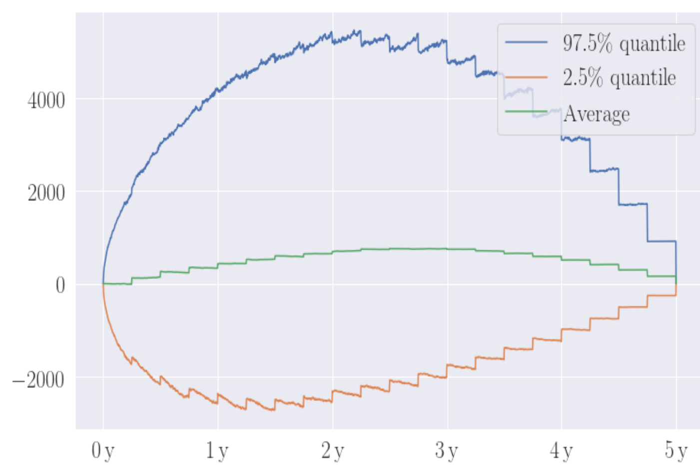

Figure 10: Mean and 2.5% and 97.5% quantiles, in basis points as a function of time, of the process $\mathrm{MtM}^\star$ in (26), calculated by Monte Carlo simulation of 5000 Euler paths of the process S. ∎

$\mathrm{MtM}^\star$ in (26).

A.2 Credit Model

For the default times τ_i of the clearing members, we use the above mentioned common shock model, where defaults can happen simultaneously with positive probabilities. The model is made dynamic, as required for XVA computations, by the introduction of the filtration of the indicator processes of the clearing member default times τ_i.

First we define shocks as pre-specified subsets of the clearing members, i.e. the singletons $\{0\}, \{1\}, \{2\}, ..., \{n\}$, for single defaults, and a small number of groups representing members susceptible to default simultaneously.

Example A.1 A shock $\{1, 2, 4, 5\}$ represents the event that all the (non-defaulted names among the) members 1, 2, 4, and 5 default at that time. ∎

As demonstrated numerically in Crépey et al. (2014, Section 8.4), a few common shocks are typically enough to ensure a good calibration of the model to market data regarding the credit risk of the clearing members and their default dependence (or to expert views about these).

Given a family $\mathcal{Y}$ of shocks, the times τ_Y of the shocks $Y \in \mathcal{Y}$ are modeled as independent time-inhomogeneous exponential random variables with intensity functions γ_Y. For each clearing member $i = 0, 1, \ldots, n$, we then set

$$\tau_i = \min_{\{Y \in \mathcal{Y};\, i \in Y\}} \tau_Y \tag{27}$$

(we recall that the default time τ of the reference clearing member corresponds to τ_0). The specification (27) means that the default time of the member i is the first time of a shock Y that contains i. As a consequence, the (pre-)default intensity γ_i of τ_i is the constant

$$\gamma_i = \sum_{\{Y \in \mathcal{Y};\ i \in Y\}} \gamma_Y.$$

Example A.2 Consider a family of shocks

$$\mathcal{Y} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1, 3\}, \{2, 3\}, \{0, 1, 2, 4, 5\}\}$$

(with $n = 5$). The following illustrates a possible default path in the model.

$t = 0.9:$	$\{3\}$	0	1	2	③	4	5	$\tau_3 = 0.9$
$t = 1.4:$	$\{5\}$	0	1	2	3	4	⑤	$\tau_5 = 1.4$
$t = 2.6:$	$\{1, 3\}$	0	①	2	3	4	5	$\tau_1 = 2.6$
$t = 5.5:$	$\{0, 1, 2, 4, 5\}$	⓪	1	②	3	④	5	$\tau_0 = \tau_2 = \tau_4 = 5.5$

At time $t = 0.9$, the shock $\{3\}$ occurs. This is the first time that a shock involving member 3 appears, hence the default time of member 3 is 0.9. At $t = 1.4$, member 5 defaults as the consequence of the shock $\{5\}$. At time 2.6, the shock $\{1, 3\}$ triggers the default of member 1 alone as member 3 has already defaulted. Finally, only members 0, 2 and 4 default simultaneously at $t = 5.5$ since members 1 and 5 have already defaulted before. ∎

We consider a CCP with $n+1=9$ members, chosen among the 125 names of the CDX index on 17 December 2007, in the turn of the subprime crisis. The default times of the 125 names of the index are modeled by a dynamic Marshall-Olkin copula model with 5 common shocks, with[2] shock intensities γ_Y calibrated to the CDS and CDO market data of that day (see Crépey et al. (2014, Sect. 8.4.3)). Table 3 shows the time

Σ_i	45	52	56	61	73	108	176	367	1053
ν_i	9.20	(1.80)	(4.60)	1.00	(6.80)	0.80	(13.80)	8.80	7.20

Table 3: (*Top*) Average 3 and 5 year CDS spread Σ_i of each member at time 0 (17 December 2017), in basis points. (*Bottom*) Swap position ν_i of each member, where parentheses mean negative numbers (i.e. short positions). ∎

0 credit spread Σ_i and the swap position ν_i of each of our nine clearing members. In particular, we have in terms of the process $\mathrm{MtM}^\star$ in (26):

$$\mathrm{MtM}^i = (-\nu_i) \times \mathrm{MtM}^\star \tag{28}$$

(sign-wise, the processes MtM^i are considered from the point of view of the CCP). We write $\mathrm{Nom}_i = \mathrm{Nom} \times |\nu_i|$.

Hereafter we denote by Φ and ϕ the standard normal cumulative distribution and density functions.

A.3 Initial Margins

We assume that the margins and default fund contribution of each clearing member are updated in continuous time[3] while the member is non-default and are stopped before its default time, until the liquidation of its portfolio occurs after a period of length $\delta =$ one week. In particular, we set

$$\mathrm{VM}^i_t = \mathrm{MtM}^i_t \text{ and } \beta_t \mathrm{IM}^i_t = \left(\mathbb{V}\mathrm{a}\mathbb{R}^{*,a_{im}}_t \left(\beta_{t^\delta}(\mathrm{MtM}^i_{t^\delta} + \Delta^i_{t^\delta}) - \beta_t \mathrm{MtM}^i_t\right)\right)^+, \tag{29}$$

for some IM quantile level a_{im}.

By (26) and (28)

$$\beta_{t^\delta}(\mathrm{MtM}^i_{t^\delta} + \Delta^i_{t^\delta}) - \beta_t \mathrm{MtM}^i_t = \mathrm{Nom} \times \nu_i \times f(t) \times (\widehat{S}_t - \widehat{S}_{t^\delta}), \tag{30}$$

with $f(t) = \sum_{l=l_{t^\delta}+1}^{d} \beta_{T_l} h_l e^{\kappa T_{l-1}}$.

Remark A.1 At least, (30) holds whenever there is no coupon date between t and t^δ (cf. Andersen, Pykhtin, and Sokol (2017)). Otherwise, i.e. whenever $l_{t^\delta} = l_t + 1$, the term $\beta_{T_{l_t}} h_{l_t} (S_{T_{l_t - 1}} - \bar{S})$ in (26) induces a small and centered difference

$$\mathrm{Nom} \times \nu_i \times h_{l_{t^\delta}} \beta_{T_{l_{t^\delta}}} \left(e^{\kappa T_{l_t}} \widehat{S}_t - S_{T_{l_t}}\right) \tag{31}$$

[2]Piecewise-constant 0–3y and 3y–5y.

[3]Instead of daily and monthly under typical market practice.

between the left hand side and the right hand side in (30). As $\delta \approx$ a few days, a coupon between t and t^δ is the exception rather than the rule. Moreover the resulting error (31) is not only rare but also small and centered. As all XVA numbers are time and space averages over future scenarios, we can and do neglect this feature in our numerics. ∎

Lemma A.1 *For $t \leq T_i$, we have*

$$\beta_t \mathrm{IM}_t^i = \mathbb{V}\mathrm{a}\mathbb{R}_t^{*,a_{im}} \Big(\beta_{t^\delta}(\mathrm{MtM}_{t^\delta}^i + \Delta_{t^\delta}^i) - \beta_t \mathrm{MtM}_t^i \Big) = \mathrm{Nom}_i \times B_i(t) \times \widehat{S}_t, \tag{32}$$

where

$$B_i(t) = f(t) \times \begin{cases} e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im}) - \frac{\sigma^2}{2}\delta} - 1, & \nu_i \leq 0 \\ 1 - e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im}) - \frac{\sigma^2}{2}\delta}, & \nu_i > 0. \end{cases} \tag{33}$$

Proof. This follows from (29) and (30) in view of the Black model used for $\widehat{S}$. ∎

A.4 CVA of the CCP

Lemma A.2 *We have, for $s \leq T$:*

$$\mathbb{E}_s^* \left[\left(\beta_{s^\delta}(\mathrm{MtM}_{s^\delta}^i + \Delta_{s^\delta}^i) - \beta_s(\mathrm{MtM}_s^i + \mathrm{IM}_s^i) \right)^+ \right] = \mathrm{Nom}_i \times A_i(s) \times \widehat{S}_s,$$

where

$$A_i(t) = (1 - a_{im}) \times f(t) \times e^{-\frac{\sigma^2\delta}{2}} \begin{cases} e^{\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}} - e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im})}, & \nu_i \leq 0 \\ e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im})} - e^{-\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}}, & \nu_i > 0. \end{cases} \tag{34}$$

Proof. In view of (29) and (30), the conditional version of the identity

$$\mathbb{E}^*[X \mathbb{1}_{X \geq \mathbb{V}\mathrm{a}\mathbb{R}^{*,a}(X)}] = (1-a)\mathbb{E}\mathbb{S}^{*,a}(X)$$

yields

$$\begin{aligned} &\mathbb{E}_s^* \left[\left(\beta_{s^\delta}(\mathrm{MtM}_{s^\delta}^i + \Delta_{s^\delta}^i) - \beta_s(\mathrm{MtM}_s^i + \mathrm{IM}_s^i) \right)^+ \right] \\ &= \mathrm{Nom} \times (1 - a_{im}) \times f(t) \left[\mathbb{E}\mathbb{S}_s^{*,a_{im}} \left(\nu_i(\widehat{S}_t - \widehat{S}_{t^\delta}) \right) - \mathbb{V}\mathrm{a}\mathbb{R}_s^{*,a_{im}} \left(\nu_i(\widehat{S}_t - \widehat{S}_{t^\delta}) \right) \right]. \end{aligned}$$

The result follows in view of the Black model used for $\widehat{S}$. ∎

Proposition A.1 *We have, for $s \leq T$:*

$$\begin{aligned} \beta_t \mathrm{CVA}_t^{ccp} = \sum_i \mathrm{Nom}_i \times \\ \left(\mathbb{1}_{t<\tau_i} \widehat{S}_t \int_t^{\bar{T}} A_i(s)\gamma_i(s) e^{-\int_t^s \gamma_i(u)du} ds \, + \, \mathbb{1}_{\tau_i < t < \tau_i^\delta} E_i(\tau_i, \widehat{S}_{\tau_i}, t, \widehat{S}_t) \right), \end{aligned} \tag{35}$$

where, setting $y^i_\pm = \frac{\ln(\widehat{S}_t/\widehat{S}_{\tau_i})}{\sigma\sqrt{\tau_i^\delta - t}} \pm \frac{1}{2}\sigma\sqrt{\tau_i^\delta - t}$,

$$E_i(\tau_i, \widehat{S}_{\tau_i}, t, \widehat{S}_t) = f(\tau_i) \times \begin{cases} \widehat{S}_t\Phi(y^i_+) - \widehat{S}_{\tau_i}\Phi(y^i_-), & \nu_i \le 0 \\ \widehat{S}_{\tau_i}\Phi(-y^i_-) - \widehat{S}_t\Phi(-y^i_+), & \nu_i > 0. \end{cases}$$

Proof. We have (cf. (12))

$$\begin{aligned}
\beta_t \mathrm{CVA}^{ccp}_t &= \sum_i \mathbb{1}_{t<\tau_i^\delta} \mathbb{E}^*_t \left[\left(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right] \\
&= \sum_i \mathbb{1}_{t<\tau_i} \mathbb{E}^*_t \left[\mathbb{E}^*_{\tau_i} \left(\left(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right) \right] \\
&\quad + \sum_i \mathbb{1}_{\tau_i<t<\tau_i^\delta} \mathbb{E}^*_t \left[\left(\beta_{\tau_i^\delta}(\mathrm{MtM}^i_{\tau_i^\delta} + \Delta^i_{\tau_i^\delta}) - \beta_{\tau_i}(\mathrm{MtM}^i_{\tau_i} + \mathrm{IM}^i_{\tau_i}) \right)^+ \right] \\
&= \sum_i \mathbb{1}_{t<\tau_i} \mathbb{E}^*_t \int_t^{\bar{T}} \mathbb{E}^*_s \left[\left(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s(\mathrm{MtM}^i_s + \mathrm{IM}^i_s) \right)^+ \right] \gamma_i(s) e^{-\int_t^s \gamma_i(u)du} ds \\
&\quad + \mathrm{Nom} \sum_i \mathbb{1}_{\tau_i<t<\tau_i^\delta} f(\tau_i)\, \mathbb{E}^*_t \left[\left(\nu_i(\widehat{S}_{\tau_i} - \widehat{S}_{\tau_i^\delta}) \right)^+ \right], \qquad (36)
\end{aligned}$$

by virtue of (30) and of the conditional distribution properties of the common shock model provided in Crépey et al. (2014, Section 8.2.1). We conclude the proof by an application of Lemma A.2 to the first line in (36) and of the Black formula to the second line. ∎

A.5 Unsecured Borrowing vs. Margin Lending MVAs

In the setup of our case study, the generic expressions in (25) for the unsecured borrowing vs. margin lending MVAs reduce to deterministic time integrals. Let $\bar{\lambda}_i = (1-R_i)\gamma_i$, where R_i is the recovery rate of the member i.

Proposition A.2 *The unsecured borrowing* MVA *of member i is given, at time 0, by*

$$\mathrm{MVA}^{ub,i}_0 = \mathrm{Nom}_i\, S_0 \int_0^T B_i(s)\bar{\lambda}_i(s) ds.$$

Proof. We set $i = 0$. By the first identity in (25) and the immersion properties of the common shock model (cf. Crépey and Song (2016)), we have

$$\mathrm{MVA}^{ub}_0 = \mathbb{E} \int_0^T \beta_s \bar{\lambda}(s) \mathrm{IM}_s ds = \mathbb{E}^* \int_0^T \beta_s \bar{\lambda}(s) \mathrm{IM}_s ds. \qquad (37)$$

Hence the result follows from Lemma A.1. ∎

Lemma A.3 *We have, for $s \geq 0$,*

$$\mathbb{E}_s^*\Big[\big(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s \mathrm{MtM}^i_s\big)^+\Big] = \mathrm{Nom}_i\, C(s)\, \widehat{S}_s, \tag{38}$$

where

$$C(t) = f(t)\left[\Phi\left(\frac{\sigma\sqrt{\delta}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{\delta}}{2}\right)\right]. \tag{39}$$

Proof. In view of (30), it comes:

$$\big(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s \mathrm{MtM}^i_s\big)^+ = \mathrm{Nom} \times f(s)\,\Big(\nu_i(\widehat{S}_s - \widehat{S}_{s^\delta})\Big)^+.$$

Hence the result follows from the Black formula. ∎

Proposition A.3 *The margin lending* MVA *of member i is given, at time 0, by*

$$\mathrm{MVA}_0^{sl,i} = \mathrm{Nom}_i\, S_0 \int_0^T \Big(C(s) - A_i(s)\Big)\bar{\lambda}_i(s)ds.$$

The blending ratio in (24) is constant and given, for $\nu_i \leq 0$, by

$$\mathrm{blend}_i = \frac{\Phi\left(\frac{\sigma\sqrt{\delta}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{\delta}}{2}\right) - (1-a_{im})e^{-\frac{\sigma^2\delta}{2}}\left(e^{\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}} - e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im})}\right)}{e^{\sigma\sqrt{\delta}\Phi^{-1}(a_{im}) - \frac{\sigma^2}{2}\delta} - 1} \tag{40}$$

and, for $\nu_i > 0$, by

$$\mathrm{blend}_i = \frac{\Phi\left(\frac{\sigma\sqrt{\delta}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{\delta}}{2}\right) - (1-a_{im})e^{-\frac{\sigma^2\delta}{2}}\left(e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im})} - e^{-\sigma\sqrt{\delta}\frac{\phi(\Phi^{-1}(a_{im}))}{1-a_{im}}}\right)}{1 - e^{\sigma\sqrt{\delta}\Phi^{-1}(1-a_{im}) - \frac{\sigma^2}{2}\delta}}. \tag{41}$$

Proof. By the last identity in (25) and the immersion properties of the common shock model, we have

$$\mathrm{MVA}_0^{sl,i} = \mathbb{E}^*\left[\int_0^T \beta_s \bar{\lambda}_i(s)\xi^i_s\, ds\right], \tag{42}$$

where, for $s \geq 0$,

$$\begin{aligned}\beta_s\xi^i_s &= \mathbb{E}_s^*\left[\big(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s\mathrm{MtM}^i_s\big)^+ \wedge \beta_s \mathrm{IM}^i_s\right] \\ &= \mathbb{E}_s^*\left[\big(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s\mathrm{MtM}^i_s\big)^+\right] \\ &\qquad - \mathbb{E}_s^*\left[\big(\beta_{s^\delta}(\mathrm{MtM}^i_{s^\delta} + \Delta^i_{s^\delta}) - \beta_s(\mathrm{MtM}^i_s + \mathrm{IM}^i_s)\big)^+\right] \\ &= \mathrm{Nom}_i\,\widehat{S}_s\big(C(s) - A_i(s)\big),\end{aligned} \tag{43}$$

by Lemmas A.3 and A.2.

The blending ratio in (24) is given by

$$\frac{\big(C(t) - A_i(t)\big)}{B_i(t)},$$

where the $f(t)$ factors simplify between the numerator and the denominator (cf. (33), (34), and (39)), yielding (40)-(41). ∎

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