

Counterparty Risk and Funding

A Tale of Two Puzzles

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Counterparty Risk and Funding

A Tale of Two Puzzles

Stéphane Crépey

University of Evry Val d'Essonne, Evry, France

and

Tomasz R. Bielecki

Illinois Institute of Technology, Chicago, USA

With an Introductory Dialogue by

Damiano Brigo

Imperial College, London, UK



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Preface

Introduction

This book is concerned with studying risk embedded in financial transactions between two parties, called the bank and its counterparty. This terminology is a useful convention used throughout the text, as, of course, the bank itself is the counterparty in the transaction relative to the bank's counterparty in the transaction. The term counterparty risk thus applies to both parties in the transaction. It will be analyzed from the perspective of the bank throughout the book.

The credit crisis and the ongoing European sovereign debt crisis have highlighted the native form of credit risk, namely counterparty risk. This is the risk of non-payment of promised cash flows due to the default by a party in an over the counter (OTC) derivative transaction. Thus, this is the risk born by a party in an OTC transaction, that its counterparty defaults on delivery of the promised payments. The value (price) of this risk was initially called credit valuation adjustment (CVA), and the volatility of this value is of interest. A key related issue, especially with credit derivatives, is the wrong-way risk, i.e. the risk that occurs when exposure to a counterparty is adversely correlated with the credit quality of that counterparty. Moreover, as banks themselves have become risky, counterparty risk must be analyzed from the bilateral perspective: not only CVA needs to be accounted for, but also debt valuation adjustment (DVA), so that the counterparty risk of the two parties to the OTC contract are jointly accounted for in the modeling. In this context the classical assumption of a locally risk-free asset which is used for financing purposes (lending and borrowing as needed) is no longer sustainable, which raises the companion issue of proper accounting for the funding costs of a position, namely the funding valuation adjustment (FVA), also called in this book liquidity valuation adjustment (LVA) when considered net of the credit spread. Another related issue is that of replacement cost (RC) corresponding to the fact that, at the default time of a party, the valuation of the contract by the liquidator may differ from the economic value of the contract (cost of the hedge) right before that time. Also since August 2007 one saw the emergence of a systemic counterparty risk, referring to various significant spreads between quantities that were very similar before, such as OIS rates versus Libor. Through its relation with the concept of discounting, this systemic component of counterparty risk has impacted on all derivatives markets. And the list goes on since people now talk about KVA for capital valuation adjustment (in reference to the capital cost of CVA volatility), or AVA for additional valuation adjustment (to reach a notion of "prudent value", integrat-

ing model risk and credit spreads including own, recently opposed to fair value by Basel). In the book we also consider RVA for a rating valuation adjustment, accounting for rating triggers. An acronym XVA was even introduced to generically refer to this increasing list of adjustments (see Carver (2013)).

All these adjustments, which are interdependent and must be computed jointly, are today among the main P&L generators of investment banks. As it is seen, dealing with these adjustments involves many areas: modeling, computation, pricing, risk management, regulation, economics, legal, lobbying, politics (even geopolitics, through varying legislative and legal practices), areas that often have conflicting objectives. The current trend of regulation is to push participants to negotiate centrally via clearing houses and to guarantee their failure through collateralization, posing a serious liquidity constraint on the market, for margin calls abound.

The basic counterparty risk mitigation tool is a Credit Support Annex (CSA) specifying the valuation scheme which will be applied by the liquidator in case of default of a party before a certain horizon T , including the netting rules which will be applied for valuing the different OTC derivatives between the two parties at that time. This “CSA value process”, i.e. the value process of an OTC contract subject to the CSA specification, is also used to define a collateralization scheme between the two parties, similar to the margining procedure in futures contracts (except that the collateral posted through a CSA typically bears some interest). However, wrong-way risk and gap risk (slippage of the value of the portfolio between the time of default, where the collateral is frozen, and the time of liquidation of the portfolio) imply that collateralization cannot be a panacea (and it also poses liquidity problems as indicated above). Therefore counterparty risk cannot be simply mitigated through collateral; it also needs to be hedged against default and/or market risk. Eventually, the collateralized and hedged portfolio needs to be funded, which takes us to the related funding issue, with the related controversial issues of windfall benefit at own default and DVA/FVA overlap or double counting.

In this book we provide an analytical basis for the quantitative methodology of dynamic valuation, mitigation and hedging of bilateral counterparty risk on OTC derivative contracts under funding constraints. We review counterparty risk in its different aspects such as CVA, DVA, FVA, LVA, RC, RVA, wrong-way risk, multiple funding curves and collateral. The book is intended primarily for researchers and graduate students in financial mathematics. In addition, it is addressed to financial quants and managers in banks, CVA desks, as well as members of the supervisory bodies.

Outline

Within the bank, every particular business trading desk has an accurate picture of only its own activity. It usually lacks the global view of the bank activities and, specifically, the aggregated data that are required to properly value the counterparty risk and funding cash

flows. Therefore in major investment banks the trend is to have central desks in charge of collecting the global information and of valuing and hedging counterparty risk, also accounting for any excess funding costs involved. We will call total valuation adjustment (TVA) the aggregate value of all the adjustments which are required in order to account for bilateral counterparty risk under funding constraints. Accordingly, we generically call TVA desk a central desk in charge of the corresponding risks (in practice it is typically split between a unit called CVA desk in charge of the counterparty risk adjustments, and the ALM (or treasury desk), which takes care of the funding issues). A “clean” price and hedge indicate a price (cost of the hedge) and hedge of promised cash flows ignoring counterparty risk and assuming that the trading strategy is funded at the OIS rate (as a typical rate of remuneration of the collateral, also the best market proxy for a risk-free rate). The price-and-hedge of a contract is then obtained as the difference between the clean price-and-hedge provided by the related business trading desk and a price-and-hedge adjustment provided by the TVA desk. Note that, since market quotes typically reflect prices of fully collateralized transactions which, as we will see in the book, can be considered as clean prices, the clean price is also the relevant notion of valuation at the stage of model calibration.

This allocation of tasks between the various business trading desks of an investment bank and a central TVA desk motivates a mathematical TVA approach to the problem of valuing and hedging counterparty risk and funding constraints. Moreover, the TVA will emerge in the book not only as a very important financial concept, but also as a useful tool in the mathematical analysis of counterparty risk subject to funding constraints. Indeed, the mathematical analysis reveals that the TVA process Θ can be interpreted as the price process of an option on the clean price P ; this option, which is called the contingent credit default swap (CCDS), pays so-called exposure at default and funding dividends.

Part I sets the financial landscape, starting with a “Galilean dialogue” touching upon most of the topics of the book. We then provide an economic analysis of the post-crisis multi-curve reality of financial markets, which is an important feature underlying several aspects of the financial and mathematical analysis of counterparty risk and funding costs. In Part II we describe in mathematical but model-free terms all the basic elements of this pricing and hedging framework. Though nice theoretically, these do not immediately lend themselves to any concrete computations. To make them practical, we therefore specify in Part III the setup for a reduced-form framework with respect to a reference filtration, in which the risk of default of the two parties only shows up through their default intensities. The above-mentioned reduced-form modeling approach results in good tractability, but is obtained at the cost of a standard immersion hypothesis between the reference filtration and the full model filtration, which is the reference filtration progressively enlarged by the default times of the two parties. From the financial point of view, such a standard immersion setup implies a weak or indirect dependence between the counterparty risk and the underlying contract exposure. In particular, it excludes the possibility of a promised dividend of the contract at the default time of a party. This is acceptable in standard cases, such as counterparty risk on interest rate derivatives. But it is too restrictive for cases of strong wrong-way risk, such as with counterparty risk on credit derivatives, which is

treated separately in Part IV by means of dynamic copula models. However, due to the high-dimensional context of credit portfolio models, we only deal here with counterparty risk, ignoring the nonlinear funding issue. Moreover, the credit portfolio models of this part only admit two possible states for each obligor: default and non-default. In Part V, we first present a credit migrations model which allows one to account for rating dependent CSA clauses. The concluding chapter then sheds a unified perspective on the different kinds of modeling employed in this book (except for the above-mentioned credit migrations model), showing how a reduced-form approach can be developed in the models of Part IV, which also opens the door to nonlinear FVA computations in credit portfolio models. Part VI is a mathematical appendix covering classical tools from stochastic analysis and a brief introduction to the theory of Markov copulas.

About the title of the book

The two “puzzles” alluded to in the title of the book¹ are (rather than counterparty risk and funding in general):

- the DVA/FVA puzzle regarding the interaction and possible overlap between DVA (own credit) and FVA (funding) terms;
- the top-down versus bottom-up portfolio credit modeling puzzle, which the CDO industry had been trying to solve for a long time and has basically failed, that also needs to be addressed in order to properly deal with counterparty risk on credit derivatives.

Regarding the first puzzle, it is shown in Part III how, once the dependence structure of the problem is understood and suitably formalized in terms of pre-default BSDEs, the DVA/FVA overlap issue can be solved. Regarding the second puzzle, we propose in Part IV dynamic copula models of portfolio credit risk and, in particular, the Markov copula common-shock model of Chapters 8 through 10. These are, in our opinion, the two major contributions of the work presented in this book. A bridge between the approaches of Part III and IV is made in the concluding chapter of Part V, where a unified perspective is given in terms of marked default times, under the so-called condition (A).

¹Cf. “A Tale of Two Cities” by Charles Dickens.

Standing Notation, Terminology and Assumptions

A time dependence is denoted in functional form by (t) when it is deterministic and as a subscript by $_t$ for a stochastic process. Somewhat unconventionally, we will frequently use notation X_t , rather than just X or $X.$, for a stochastic process $(X_t)_{t \geq 0}$. Regarding operators, a subscript (if any) indicates the variable in which the action takes place, e.g. ∂_t for time derivation, or \mathcal{A}_x for an operation acting in the direction of x .² By default, a real valued function of real arguments is assumed Borel measurable; all price processes are assumed to be special semimartingales and all semimartingales (including finite variation processes) are taken in a càdlàg version; “martingale” means local martingale, but the strict martingale property is assumed whenever necessary; all inequalities between random quantities are to be understood almost surely or almost everywhere, as suitable; all the cash flows are assumed to be integrable; all the market quotes (calibration data) are denoted with a star to distinguish them from their model counterparts (model clean prices as explained in the Outline above), which are denoted by the same letter without a star.

The symbols τ_b and τ_c represent the default times of the bank and of the counterparty respectively; $\mathbb{H}^b = (\mathcal{H}_t^b)_{t \in [0, T]}$ and $\mathbb{H}^c = (\mathcal{H}_t^c)_{t \in [0, T]}$ stand for the natural filtrations of τ_b and τ_c , that is the filtrations generated by the corresponding indicator processes, i.e. $\mathcal{H}_t^i = \sigma(\tau_i \wedge t) \vee \sigma(\tau_i > t)$, for any time t and $i = b$ or c ; $|S|$ is the cardinality of a finite set S ; \mathbb{R}_+ denotes the nonnegative half-line $[0, +\infty)$; \int_a^b is to be understood as $\int_{(a, b]}$ (so in particular $\int_a^b = 0$ whenever $a \geq b$); $x^+ = \max(x, 0)$ and $x^- = \max(-x, 0) = (-x)^+$ are the positive and the negative parts of x a real number x (sometimes, in case there is a superscript as in x^e , we write $x^{e, \pm}$ instead of $(x^e)^\pm$); δ represents a Dirac measure; $^\top$ stands for transpose of a matrix.

Bibliographic Guidelines

Here are the main sources for this book:

- for Part I: Crépey and Douady (2013a, 2013b) (Chapter 2); the Galilean dialogue in Chapter 1 is essentially original material renewing the Platonic dialogue of Brigo, Morini, and Pallavicini (2013);
- for Part II: Bielecki and Crépey (2013) (Chapter 3); Crépey (2012a) (Chapter 4);
- for Part III: Crépey (2012b) (Chapter 5); Crépey, Gerboud, Grbac, and Ngor (2013) (Chapter 6);

²In this we depart from the standard operator theory notation where a subscript t typically emphasises a time-inhomogeneity of the operator \mathcal{A} . So, in our case, any such time-inhomogeneity is implicit in \mathcal{A} (or \mathcal{A}_x).

- for Part IV: Crépey, Jeanblanc, and Wu (2013) (Chapter 7); Bielecki, Cousin, Crépey, and Herbertsson (2013c,2013a, 2013b) and Bielecki and Crépey (2013) (Chapter 8); Crépey, Jeanblanc, and Zargari (2010) and Bielecki, Crépey, Jeanblanc, and Zargari (2012) (Chapter 9); Assefa, Bielecki, Crépey, and Jeanblanc (2011) and Crépey and Rahal (2013) (Chapter 10);
- for Part V: Bielecki, Cialenco, and Iyigunler (2013) (Chapter 11); Crépey (2014) (Chapter 12);
- for Part VI: Sect. 3.5, 4.2 and 4.3.2 in Crépey (2013) (Chapter 13); Bielecki, Jakubowski, and Niewęglowski (2013,2012) and Bielecki, Jakubowski, Vidozzi, and Vidozzi (2008) (Chapter 14).

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Part I

Financial Landscape



Chapter 1

A Galilean Dialogue on Counterparty Risk, CVA, DVA, Multiple Curves, Collateral and Funding

In this introductory chapter, which is written in the form of a Galilean dialogue renewing the Platonic dialogue of Brigo, Morini, and Pallavicini (2013), we present the financial landscape of the book, touching upon credit value at risk (credit VaR), potential future exposure (PFE), expected exposure (EE), expected positive exposure (EPE), credit valuation adjustment (CVA), debt valuation adjustment (DVA), CVA versus DVA hedging, close-out conventions, netting clauses, collateral modeling, gap risk, rehypothecation, wrong-way risk, Basel III, inclusion of funding costs, first-to-default risk, contingent credit default swaps (CCDS), CVA restructuring possibilities through margin lending, backward stochastic differential equations (BSDEs) and dynamic copulas (Markov copulas in particular).

The dialogue is in the form of a conversation among three friends respectively working as a quant (Salva), a trader (Sage) and a regulator (Simeone), in the manner of *Dialogo sopra i due massimi sistemi del mondo* (Galileo Galilei). Quoting Wikipedia: “The book is presented as a series of discussions, over a span of four days, among two philosophers and a layman: Salviati, from Florence, argues for the Copernican position and presents some of Galileo’s views directly. [...] Sagredo, from Venice, is an intelligent layman who is initially neutral. [...] Simplicio, a dedicated follower of Ptolemy and Aristotle, presents the traditional views and the arguments against the Copernican position”.

In our case:

Sage - Sagredo has been a trader for most of his career, and recently joined the top management of a tier-1 investment bank. He has a general and utilitarian grasp of maths. He believes in global growth, developed economy and he is a capitalist at heart. He is a derivatives and more generally a market enthusiast. He has a master’s from Chicago. He is based in London.

Simeone - Simplicio is a US Central Bank employee, he is very attentive to regulation. He has a very prudent stance on financial innovation. He is in line with Chicago school economics, he worked briefly as model validator previously. He has a good knowledge of academic literature in economics and finance. He has a master’s from Paris, following education in a grand école. He is momentarily based in Paris at the time of the dialogue but often travels to London by train.

Salva - Salviati is a quant who worked in a variety of roles, from Front Office (rates, Credit) of investment banks to Risk Management of commercial banks. She is open minded and gently opinionated. She tries to mediate the extreme positions of the other

two and is somehow a skeptic. She has a PhD in a university of a marginal (from the corporate/anglo saxon centric point of view) country. She is also based in London.

The three characters were in touch at the time of their undergraduate studies.

At this point it is fair to stress that the characters do not have an exact correspondence with Galileo's characters. In Galileo's dialogue, roughly speaking, SIM is the bad guy, SAL is the good guy, and SAG is an enlightened layman/billionaire siding with SAL. Here we are more in a mixed situation, and while we retained the three-person scheme and a formal name mapping, we would discourage³ a full identification. For example, you'll notice that our SIM comes out much better than Galileo's SIM, whereas our SAG comes out definitely worse than Galileo's. A similar remark applies to the following section, "to the discerning reader", that should not be interpreted literally.

1.1 To the Discerning Reader

Several years ago¹, there was published in Basel a powerful edict which, in order to obviate the dangerous tendencies of our present age, imposed a seasonable silence upon the Pythagorean opinion that counterparty risk, collateral and funding pricing cannot be standardized. There were those who asserted that this decree had its origin not in judicious inquire, but in passion none too well informed. Complaints were to be heard that advisers who were totally unskilled at financial observations ought not to clip the wings of reflective intellects by means of rash standardization and regulation. In reaction to the Pythagorean view, the Basel supporters impudently accused the standardization critics of lack of a global vision.

Upon hearing such carping insolence, my zeal could not be contained. Being thoroughly informed about that prudent determination, I decided to appear openly in the theater of the world as a witness of the sober truth.

I was often to be found in the marvelous city of London, in discussions with Signore Sage, a man of notable wealth and trenchant wit. From London came also Signora Salva, a sublime intellect who fed no more hungrily upon good wine than it did upon fine meditations. I often talked with these two of such matters in the presence of Signor Simeone, a Central Banker whose interpretations of Basel I, II and III always struck me by their depth.

Now, since diverging destinies have deprived London of those three great intellects, I have resolved to make their fame live on in these pages, so far as my poor abilities will permit, by introducing them as interlocutors in the present argument.

It happened that several discussions had taken place casually at various times among them and had rather whetted than satisfied their thirst for learning. Hence very wisely they resolved to meet together on certain days during which, setting aside all other business,

¹Refashioned after the version translated by Stillman Drake and annotated and condensed by S. E. Sciortino.

they might apply themselves more methodically to the contemplation of the economics wonders of the markets. They met in a pub in London; and, after the customary but brief exchange of compliments, Salva commenced as follows.

1.2 The First Day

1.2.1 General Introduction, Size of Derivatives Markets, Exposures, Credit Var, Basel

SALVA. Good to see you, Sage, good to meet you again, Simeone. It's good to meet after such a long time.

SAGE. Indeed, it has been a while. I find you well, you have not lost an ounce of acumen, beauty and charme.

SALVA. Flattery will get you anywhere. [smiles]

SIMEONE. You both look well.

SAGE. What have you been doing since university? After my master in Chicago I started a career as a junior quant in a trading desk, working on rates and FX. Then I moved on to trading, first with a junior role and then more and more senior. I traded in the CDO correlation desk of Top Bank, then moved to Super Bank and in the last four years I have taken charge of the CVA trading desk of Platinman Bank.

SALVA. You must have made quite a lot of money.

SAGE. Not as much as I'd love to [laughs] and I lost a lot of it too You know the saying, if you still need to work it means you haven't made enough money.

SALVA. Well, the last few years have been tough on a lot of people. I also started as a quant, but four years later than you, since I had to complete a PhD. I have stayed in the quant space, as I have worked on modeling of credit risk and derivatives pricing and hedging more generally, first for the front office of an investment bank and then for the risk department of a commercial bank. Now I am taken between departments as I coordinate efforts for a consistent CVA modeling platform for the bank.

SAGE. Not a bad idea, since I heard you had two babies? Congratulations, by the way. I guess the risk office is becoming quite an environment for quants, although I noticed

the front office keeps being the most sought-after destination for quants, especially young ones.

SIMEONE. These quants would work on derivatives... that business that in 2011 reached a notional of more than 700 trillion USD\$?

SAGE. Well that is according to the Bank of International Settlements, that bunch of bureaucrats

SALVA. But you have to admit these numbers can dwarf imagination. 700 trillion... $7 \cdot 10^{14}$, or more prosaically 700 000 000 000 000 USD.

SAGE. Never traded on those notionals, though; I wish I had. [laughs]

SIMEONE. You are not funny. You know what was the nominal GDP of US in 2011? About 15 trillion... and Europe was slightly above 17 trillion. The world GDP was about 70 trillions. Keeping this in mind, 700 trillion for derivatives sounds like madness, even when discounting double counting etc. Maybe we bureaucrats sometimes look at the big picture rather than being obsessed with our personal fortune.

SAGE. Now you can't say that. I eat what I kill! You don't.

SIMEONE. Strictly speaking you don't kill the cows whose meat you eat. Nor do you produce the electricity you consume. Nor...

SALVA. Now, slow down you both!! Testosterone is overrated anyway. I still think that derivatives are useful, ok? and are not bad in themselves. As many other objects, they can be used well or abused, their use is not implicit in their definition.

SAGE. Well spoken.

SIMEONE. I am not convinced. Do you find it sensible that the derivatives market can grow to be 10 times as large as the GDP of the planet? It seems madness to me.

SAGE. Why? Nobody pointed a gun and forced the banks to trade derivatives.

SALVA. Mmmhhhh I don't know about that... it looks like a game that got out of hand at some point and nobody could stop...

SAGE. You can't stop progress, you need to keep growing and accelerating, going back does not help.

SALVA. Slowing down can be helpful occasionally, you know [smiles]. And I am not sure

about this “perennial economic growth” idea anymore. On the other hand, you studied at Chicago and I recently became a responsible mum [smiles].

SIMEONE. Well, there is also the economic cycle; it is not always growth, only in the long run. You can’t deny that the industrial revolution and technological progress and free trading lead to growth.

SALVA. ... and to depletion of natural resources, pollution, global warming and a lot of developed economies on the verge of default, if you look at the data.

SAGE. I think the average person lives better and healthier today than 100 years ago.

SALVA. The real question is how long is this sustainable? All developed economies are quite in trouble with this “crisis”... is it really a crisis? A crisis has a turning point, it starts looking more like a chronic syndrome.

SIMEONE. This is not unprecedented, look at the great depression.

SAGE. You look at it! It is already bad enough to hear all the bad news...

SALVA. Ok that’s quite enough, you two!! [laughs gently] Simeone, what have you been up to? You seem to know the big picture pretty well.

SIMEONE. After I finished my master’s in London I worked in model validation for a bank and then moved to regulation. I joined the Central Reserve Bank in Metropolis, in the States, five years ago, to work on counterparty risk.

SAGE. That is definitely one of the safest places where you can be now. It’s funny that now we are all connected to counterparty risk.

SALVA. I guess that has not made you popular with fellow traders in other desks!!

SAGE. You don’t know the half of it... these xxxxxxxx don’t have a clue of the risks we cover for them and don’t want to be charged a penny.

SIMEONE. They may feel that you are not really helping them. You charge them a fee that should then be used to hedge counterparty risk, funding costs, etc, but can you really hedge such risks effectively? I don’t think you can, so perhaps these colleagues of yours feel they are wasting their money and that if the bad day comes when a counterparty to their deal defaults, you won’t really protect them in any relevant way.

SALVA. That’s a little harsh. I have seen the same attitude but this might be more a

reluctance to give up part of the P&L rather than lack of trust of the CVA desk. After all, what trader is happy to be charged additional costs?

SAGE. Yeah, that's right.

SALVA. That's not the whole story, though ... in fact, difficulties in hedging CVA and DVA are objective. You are an experienced trader. Do you think CVA can be hedged? And DVA? Can you hedge CVA and DVA just with sensitivities just because you can hedge an equity call option in Black- Scholes like that? How do you hedge wrong-way risk, recovery risk, jump to default risk? And what about even credit risk, for those counterparties that do not have a liquidly traded CDS? You can't really hedge any of those risks properly. What is the point of implementing cross-gamma hedging if you have an uncertainty of 20 percent on the recovery? And the fun one would be proxy hedging. ... who mentioned that? Goldman Sachs I think.

SAGE. Whose side are you on??? Goldman Sachs are simply the best, so if they say that...

SALVA. Right, right. As the resident anti-establishment nuts, I need to remind you of: Greek debt camouflage operations, going long or short California bonds depending on what is more convenient (not always to the client, understatement of the year), the resignation letter of Greg Smith, continued conflict of interest and public/government positions: Henry Paulson, Mario Draghi, Mario Monti, Mark Patterson... you are a big boy, so you read the newspapers, don't you? [smiles]

SAGE. [shakes head] Give me a break, next you will start a conspiracy theory... you need to read more credible sources.

SALVA. You know, I didn't find this in some obscure revolutionary magazine. As I said, it is in mainstream newspapers (New York Times anyone?) and even in wikipedia. [laughs]

SAGE. Ah, then it must be right. [laughs]

SALVA. I am simply trying to be intellectually honest here! [pretends to be offended]

SAGE. [pretends to be offensive] Are you sure?

SIMEONE. If you two are quite finished with the cabaret, I would like to say something as well. Going back to valuation and hedging of CVA and proxy hedging in particular, you should both make up your mind: is CVA a proper concept or not? Should it be pursued? Can it be hedged? If not, does it make sense as a price? I personally think it is not helpful; it should not be traded and it cannot be hedged.

SAGE. [confident] Of course it is.

SALVA. You cannot always pretend to have easy and simple answers. CVA-DVA-FVA is a very complex concept that requires very sophisticated computational techniques to be addressed properly and even defining it properly is a headache [runs hand along luminous light-brown hair]

SIMEONE. [didactically] Quantities and products that cannot be managed properly and responsibly should be banned. Imagine if you cannot even define them properly...

SAGE. [rolling eyes] Come on guys, CVA is relatively simple: your counterparty may default, thus causing you a loss if you were a creditor at the time.

SALVA. There are a lot of choices to be made when computing CVA, both on the models to be used and on the type of CVA to be computed. There are choices to be made on whether it is unilateral or bilateral (does DVA make sense?), on the closeout formulation, on how you account for collateral and rehypothecation, on whether you include first to default and on how you account for funding costs and so on. Due to the variety of possible different definitions of CVA and of modeling choices, there appears to be material discrepancies in CVA valuation across financial institutions, as pointed out in the article by Watt (2011). And you think it is simple? I do not go as far as Simeone, but I am definitely not as optimistic as you are.

SAGE. [shrugs] If you let complexity paralyze you, you will never do anything in your life. We must go as far as we can with the analysis in a reasonable time, but then we should take action. That is what a CVA desk is about.

SALVA. I think we are looking at different time horizons. Your philosophy may be ok for very short terms, but over long terms it can be quite different.

SIMEONE. Salva is right. Even your performance as a trader is measured annually with a bonus and then you may be gone. However, the consequences of your trades may affect the bank over the next 10 or 20 years.

SAGE. You know too well that if I trade poorly my reputation is ruined and I will have difficulties in finding a good position again. [mimicks gun with fingers] So it's not like bad performances are instantly forgotten.

SALVA. I am not sure about that. [laughs]

SIMEONE. As a regulator I have seen all cases happen, but there is indeed memory in the market.

SALVA. Most of the traders I know are gossip oriented, present company excepted of course, so you may be right. Gossip, however, is hardly the most reliable source of information.

SAGE. As a trader your P&L speaks for you, and I thought women were more gossip oriented. [grinning]

SIMEONE. As if mark to market were an exact science.

SALVA. [higher voice pitch] “Book this with flat volatilities, book this with bid, book this with ask, book that with the smile, book that with. . .”

SAGE. Yeah, I bet you read that too in Wikipedia. . . [shakes head]

SALVA. No I actually heard it from our traders. [laughs]

SIMEONE. Would you mind remaining serious for a minute?

SAGE. Why don't you talk to us about your regulator's job? And do you think all this exposure / PD / LGD / ED and Credit VaR stuff from Basel I, II and now III has been helping really?

SIMEONE. [didactically] Let us be careful here, as we are mixing a number of different notions. Let me cite Canabarro and Duffie (2004) from my graduate studies. This paper contains basic prototype definitions for exposure that are not always in line with the market, but can be used as a reference. First, counterparty exposure at any future time is the larger between zero and the market value of the portfolio at that time. Current exposure (CE) is the current value (say now, at time 0) of the exposure to a counterparty, so it is the current value of the portfolio if positive, and zero otherwise. It is the expectation under the pricing measure \mathbb{Q} of future cash flows, each discounted back to time 0 (now) and added up, if positive and zero otherwise. [loosens tie] Potential future exposure (PFE) at a given time is the maximum of exposure at that time under a high degree of statistical confidence. For example, the 99% PFE is the level of potential exposure that is exceeded with only 1% \mathbb{P} -probability. PFE plotted against time is the potential exposure profile, up to the final maturity of the portfolio of trades with the counterparty. I think our friend Salva can explain to us this measures issue (\mathbb{P} and \mathbb{Q}); during my master's I studied it in terms of risk premia, but maths people managed to do it much more complicated. [drinks]

SALVA. [adjusting skirt] Is that so? Well even among “quants” there is a lot of mileage there. . . some think there is only \mathbb{Q} , some think there is only \mathbb{P} and so on. . . But let me comment on the following first: PFE is typically obtained through simulation. For each future time, the price of the netting set of trades (the relevant portfolio) with a given

counterparty is simulated. A \mathbb{P} -percentile of the distribution of exposures is chosen and this is the PFE at the future time. The maximum of PFE over the life of the portfolio is called maximum potential future exposure (MPFE).

SAGE. Bla bla bla. . . do we really need all this technobubble?

SALVA. [flashing an ironic look] It depends on what you are trying to achieve of course.

SIMEONE. As a regulator I can tell you that banks care about such quantities because PFE and MPFE are usually compared with credit limits in the process of deciding whether a trade is permissible or not. I am sure you know about this. We also have expected exposure ($EE(t)$), the expectation of the exposure at a future date t under the \mathbb{P} -measure conditioned on the first to default occurring at time t . The curve of $EE(t)$ plotted against time t is the expected exposure profile. Expected positive exposure (EPE) is the time average of EE up to a given future time.² Finally, we have exposure at default (ED). This is naturally defined as the exposure value at the (random future) time of default for the counterparty.

SAGE. All right, but we are talking about default risk here and I haven't heard anything about default yet, although you mentioned twenty different definitions of exposure. . . [stifles a yawn] Why are we not mentioning any default probabilities, recoveries, etc?

SIMEONE. Indeed, in exposure there is no default simulation involved. The future portfolio value is simulated, but not the default of the counterparty. With exposure we deal with this matter: IF default happens, what is going to be our loss due to that default? So in a way, assuming default happens, we check what would be the loss in that case.

SAGE. [removing jacket] But this is not Credit VaR, is it?

SALVA. No, indeed. With Credit VaR we address the problem: what is the loss level that is not exceeded with a given \mathbb{P} probability, over a given time horizon? This does involve the inclusion of the default event of the counterparty as we generate the loss and the default probability of the counterparty and also the recovery rates are key here. But I would let our regulator colleague talk about Credit VaR

SIMEONE. [in a didactical tone, removing jacket, then pinching chin] Credit VaR is basically an attempt to measure counterparty default risk for capital requirements. It tries to measure the risk that one bank faces in order to be able to lend money or invest towards a counterparty with relevant default probability. The bank has to cover that risk by re-

²In Chapter 3 we discuss a somewhat different notion of EPE which, in particular, is considered under the pricing measure \mathbb{Q} . See also the remark 3.2.11 in this regard.

serving capital, and how much capital can be decided after the risk has been measured. A popular such measure is precisely Value at Risk (VaR), which is a quantile on the loss distribution associated with the position held by the bank, over a given time horizon. More precisely, it is a percentile (say the 99.9 percentile) of the difference between the initial value of the position and the future value of the position at the risk horizon, across scenarios. When applied to default risk, leading to Credit VaR, the horizon is usually one year. If one chooses the 99.9-th percentile, we have the loss that is exceeded in only 1 case out of 1000. There are a few different definitions, more precisely. For example, Credit VaR can be either the difference of the percentile from the mean, or the percentile itself. There is more than one possible definition. [sweats slightly, drinks, looks at Salva]

SALVA. [adjusting long hair with left hand] We may also mention that in general VaR is not a universally good measure. It has often been criticized for not being sub-additive. In a way it does not always acknowledge the benefits of diversification. In some peculiar situations the risk of a total portfolio can be larger than the total of the risks in single positions. Another measure used to be preferred, namely expected shortfall, also known as tail VaR, conditional VaR, etc. This is the expected value of the losses beyond the estimated VaR percentile. However, expected shortfall is not elicitable, as discussed in Ziegel (2013), whereas VaR is. There are other good reasons why expected shortfall doesn't work and also why coherent risk measures, ten years later, turned out not to be such a great idea after all, so the choice is not so clearcut³ [pauses a minute, sips drink] However, a large part of the industry still uses Credit VaR. This is obtained through a simulation of the basic financial variables affecting the portfolio and of the default of the counterparty before one year. The simulation is done under the historical probability measure, the \mathbb{P} we mentioned earlier, up to the risk horizon. This simulation does include the default of the counterparties. At the risk horizon, the portfolio is re-priced in every single simulated scenario. For example, if the risk horizon is one year, we then have a number of scenarios for the value of the portfolio in one year. In each scenario, if the counterparty has defaulted, we check the value of the portfolio. If this is positive, all is lost except a recovery and this enters our loss. If it is negative or zero, nothing is lost. If there has been no default up to one year in that scenario, again nothing is lost (due to default risk). After we analyze this in every scenario at one year, a distribution for the portfolio losses at the horizon is assembled based on these scenarios of portfolio values and defaults. [leans forward] Notice that by saying "priced" above, we are stating that the discounted future cash flows of the portfolio after the risk horizon are averaged conditional on each scenario at the risk horizon itself. However, this averaging happens under the pricing probability measure \mathbb{Q} and not under the historical measure. [pauses, sips drink]

SAGE. What is it with this confusion of probability measures?

³A risk measure is elicitable if it is possible to verify and compare competing estimation procedures for this measure.

SIMEONE. I warned you. . .

SALVA. [sighs, sits back] OK, suppose your portfolio is an equity forward that is traded with a client, with a final maturity of five years. To obtain the Credit-VaR you simulate equity under the \mathbb{P} measure up to one year, obtain several one year equity scenarios. You also simulate the default of the counterparty up to one year, so that you know in each scenario whether the counterparty has defaulted or not. This is again under the measure \mathbb{P} . It may be important to include “correlation” between counterparty default and equity, leading to wrong-way risk (WWR). [leans forward] Summing up: in the first part we simulate under \mathbb{P} because we need the risk statistics in the real world, under the physical probability measure. After that, in the second part, in each one-year scenario, if the counterparty has defaulted then there is a recovery value and all else is lost. Otherwise, we price the equity forward over the remaining year. However, this price is like taking the expected value of the forward in five years, conditional on each equity scenario in one year. Since we are doing valuation, these last expected values will be taken under the pricing measure \mathbb{Q} and not under \mathbb{P} . [pauses, sips drink, checks mobile] Excuse me, just checking the baby sitter messages.

SAGE. [sympathetically] Take your time.

SALVA. All right. In the first part, it may be difficult to obtain the statistics needed for the simulation. For example, for default probabilities, often one uses probabilities obtained through aggregation methods or proxying, such as the probability associated to the rating of the counterparty. This is not accurate. But by definition default of a single name happens only once, so its \mathbb{P} probability cannot be estimated in any direct way. About \mathbb{P} and \mathbb{Q} , typically default probabilities under \mathbb{Q} (which are usually obtained from CDS or corporate bond prices) are larger than those under \mathbb{P} . For example, a comparison of the \mathbb{P} and \mathbb{Q} loss distributions involved in Collateralized Debt Obligations (CDO tranches) is considered in Torresetti, Brigo, and Pallavicini (2009).

SAGE. I need a refresher, I studied VaR way back in Chicago but I am not up to speed.

SALVA. A basic intro is the book Jorion (2006), whereas for a higher technical level you have books like McNeil, Frey, and Embrechts (2005), which may, however, be too advanced for you, no offence meant [smiles]

SAGE. [fascinated] None taken, my beautiful professor!

SALVA. [smiling] From an historical perspective it is interesting to look at the original Credit VaR framework in the original “Credit Metrics Technical Document” by Gupton, Finger, and Bathia (1997). I am sure you all came across that at some point.

SAGE. I suddenly realized we haven't talked about CVA, but only Credit VaR. Why don't we discuss CVA, since that seems to be our main activity, even if from different points of view. Why don't we have an open minded discussion on this subject from different angles in front of a few drinks? We can do that tomorrow evening if you are ok with that, it would be fun I think.

SALVA. Yeah, why not?

SIMEONE. I will be there, I can prepare an agenda

SAGE. Again the bureaucrat... Why don't we keep it informal?

SALVA. I guess an agenda will help us stay focused and avoid more personalized confrontation (look at today!).

SAGE. You think we have been confrontational? And you said you worked with traders? Man, you don't know the half of it.

SALVA. You said that already. [smiles feigning boredom]

SAGE. [magnanimously] "Repetita iuvant".

SIMEONE. Here, you see why we need an agenda? On one side I feel like hugging you both after all these years, on the other hand I would like to kick Sage out of this pub, push him down the stairs, kick him again, shoot and kiss Salva and then go all together for a beer anyway.

SALVA. All right, all right. Thanks guys; see you tomorrow. Same time, ok? [adjusts skirt, pushes chair back, stands up and takes purse]

SAGE. Good for me. [checks mobile, updates calendar, stands up, kisses Salva's cheeks, shakes hands with Simeone] See you tomorrow.

SIMEONE. Excellent. [stands up, shakes hands with Sage, kisses Salva's cheeks]

1.3 The Second Day

1.3.1 CVA, DVA, Pricing, Arbitrage Free Theory, Closeout. And the Data? Ratings?

SALVA. Hello Sage, let's sit outside. The weather is quite pleasant with this sunny spring we are having.

SAGE. Hi, good idea Salva. Let me get a couple of drinks while we wait for Simeone. What will you have?

SALVA. A glass of Valpolicella please.

SAGE. Here you are, Salva.

SALVA. Thank you. So how did the trading go today?

SAGE. Today I mostly struggled through meetings with the treasury and the risk management. They are all pushing for a different system for calculation and charge of funding costs. A political nightmare.

SALVA. I imagine, since you are dealing with something that is quite P&L sensitive. [rolls the chalice in left hand]

SIMEONE. Hello Colleagues, let me get a drink and I will join you.

SAGE. Hi Simeone, great, we'll wait for you before saving the world. [laughs]

SALVA. [smiling] Yes we have big plans.

SIMEONE. Uff. . .

SAGE. First of all let me celebrate our reunion: Cheers! [raises wine glass]

SIMEONE. Cheers!

SALVA. Salute! [sipping the wine] So Sage was telling me the sort of political nightmare CVA is for his bank. It is the same where I work, but I am less impacted, being a quant. However, let's start by checking if we agree on what is CVA.

SAGE. All right, best way to explain is like this. Suppose you can trade a product either

with a default-free counterparty or with the defaultable one. Having a choice, you would always take the default-free one. This means that in order to do the trade with the risky one you require additional compensation. In other terms, you require a reduction in the product price. This reduction is exactly the credit valuation adjustment, or CVA.

SIMEONE. In other words we can define it as the difference between the price of the product without default risk and the price of the product with default risk of the counterparty included.

SAGE. That's what I said!

SALVA. Good enough for a start. So the key point here is that this is a reduction in price, or in other terms it is itself a price. As a price it is a risk neutral or pricing measure expectation of future discounted cash flows. This is an expected value at valuation time, typically now, i.e. 0, of the discounted future cash flows involved with the default risk on the specific portfolio. While Credit VaR was a \mathbb{P} measure percentile of the loss due to counterparty default at the risk horizon, this is a price at time 0, i.e. a \mathbb{Q} expectation given the information today.

SAGE. Again with this \mathbb{P} and \mathbb{Q} , we can't let you get away with this anymore. Please explain. [lights a cigar]

SALVA. [leaning back disapprovingly] You took up smoking cigars now?

SAGE. [innocently] After you do your first million it becomes compulsory, you know. It's written in Basel. [laughs] What, I hope you are not a health nut!

SIMEONE. You can smoke as much as you like as long as you puff that smelly smoke away from me. If you want to harm yourself it is none of my business.

SAGE. Pfffff.

SALVA. ... and from me, my hair and my clothes. I am sure you know how persistent that smoke smell is.

SAGE. [feigning offense, pushes chair back a little] Pfffff. This is perfume, not smell. And we are in the open. Why can't a guy smoke his cigar in peace after a hard day of work?

SALVA. [smiling ironically] All right, forgive us for not being in the unhealthy millionaires club...

SAGE. Ahahah, you know you are the only friends I have who are under one million? [puffs his cigar further]

SALVA. It would be fun if this conversation was intercepted and published by a journalist.

SIMEONE. Stop the cabaret and focus on the topic at hand.

SAGE. “Focus on the topic at hand”?? What ridiculous nonsense is that? Don’t talk to me like that. I have known you too long. [puffs]

SALVA. All right, all right. \mathbb{P} and \mathbb{Q} then. Where do I start from? Ok, statistics of random objects such as future time losses depend on the probability measure being used. Clearly, under two different probability measures the same random variable will usually have two different expected values, variances, percentiles, etc. The probability \mathbb{P} , the historical or physical probability measure (also known as real world measure), is the probability measure we use to do historical estimation of financial variables, econometrics, historical volatility calculations, historical correlations, autocorrelations, maximum likelihood estimation, etc. As we compute VaR, for example, when we simulate the financial variables up to the risk horizon, we do it under \mathbb{P} . Also, when we try to make a prediction of future market variables through economic forecast, technical analysis etc we do it implicitly under \mathbb{P} . This is because risk measurement and prediction are interesting under the statistics of the observed world. However, if we are rather trying to price an option or a structured product in a no-arbitrage framework, the no-arbitrage theory states that expected values of future discounted cash flows are to be taken under a different probability measure, namely \mathbb{Q} . These two measures are connected by a mathematical relationship depending on risk aversion, or market price of risk. In simple models, the real measure \mathbb{P} expected rate of return is given by the risk-free rate plus the market price of risk times the volatility. Under \mathbb{P} the expected value of the rate of return of an asset is hard to estimate, whereas under \mathbb{Q} one knows that the rate of return is the risk-free rate. Arbitrage-free theory says that dependence on the \mathbb{P} rate of return can be hedged away through replication techniques. And if you think about it, this is why derivatives markets have exploded. The price of a derivative does not depend on your perception of the actual (\mathbb{P}) expected future return. Actual expected returns of assets are very difficult to estimate or predict and rightly so; otherwise we would all be very rich. But to price a derivative depending on the growth of such assets you do not need to know such (\mathbb{P}) returns, in principle.

SIMEONE. You seem to imply that the creation of derivatives markets was a good development.

SAGE. Again with this story? Derivatives are here and are here to stay. How can an airline hedge fuel risk without an oil swap? Give up this nerd attitude, take note and move on! [puff]

SIMEONE. [indignant] What if you stopped and thought about something for more than three seconds?

SALVA. [waving hands] Ok, ok, calm down you guys, I told you that testosterone is overrated. Let's go back to CVA. [adjusting hair] But before we do that we need a detour through a city in Switzerland.

SAGE. And I bet it's not Zurich and starts by "B". [laughs, puffs]

SALVA. And it's not Bern [anticipating Simeone's protest] ok? Basel: 1 2 3. . .

Ok, seriously, . . . "Basel" is a set of recommendations on banking regulation issued by the Basel Committee on Banking Supervision. We are mostly interested in the second critical set of such recommendation, termed Basel II, and in the third one – Basel III. Basel II was first issued in 2004 and updated later on, to create a standard that regulators can use to establish how much capital a bank has to set aside in order to cover financial and operational risks connected to its lending and investing activities. Often banks tend to be willing to reserve as little capital as possible when covering risks, so as to be able to use the remaining funds for their activity and to have more liquidity available. The capital requirements concern overall the three areas of credit - or counterparty - risk, market risk and operational risks. The counterparty risk component of capital requirements can be measured at three different levels of increasing complexity, the "standardized approach", the foundation internal rating-based approach (IRBA) and the advanced IRBA. The standardized approach is more conservative and is based on simple calculations and quantities, so that if a bank follows that approach it is likely to find higher capital requirements than with the IRBA's. Obviously this is an important incentive for banks to develop internal models for counterparty risk and credit rating. Still, the ongoing credit crisis is generating a lot of doubt and debate on the effectiveness of Basel II and of banking regulation more generally. Basel regulation is currently under revision in view of a new set of rules commonly referred to as Basel III (later). Basel I, II and III have been heavily criticized also from a methodological point of view; see for example Blundell-Wignall and Atkinson (2010).

SIMEONE. So, summarizing, we mentioned above two main areas: (i) counterparty risk measurement for capital requirements, following Basel II, and the related Credit VaR risk measure, or (ii) counterparty risk from a pricing point of view. Basel II deals with (i) mostly and hence with concepts such as Credit VaR. I have been quite happy with that and with the efforts to improve the framework. Where I have problems is with (ii). In (ii) we are updating the value of a specific instrument or portfolio, traded with a counterparty, by altering the price to be charged to the counterparty. This reduction in price accounts for the default risk of the counterparty. As Sage said earlier, all things being equal, we would always prefer entering a trade with a default-free counterparty than with a default risky one. Hence we charge the default risky one a supplementary amount besides the default-free cost of the contract, that is thus reduced to us exactly of that amount. This

is often called credit valuation adjustment, or CVA. As Salva explained earlier, since it is a price, it is computed entirely under the \mathbb{Q} probability measure, the pricing measure. \mathbb{P} does not play a role here. We are computing a price, not measuring risk statistics. As I mentioned earlier, [looks around skeptically] I have doubts this notion is helpful or even appropriate. Despite this, it has been there for a while; see for example Duffie and Huang (1996), Bielecki and Rutkowski (2001), Brigo and Masetti (2005).

SALVA. However, it became more and more important after the 2008 defaults. I recall that in 2002 not many people cared about this.

SIMEONE. Now perhaps Salva could summarize how this CVA term looks from a modeling point of view, so that we can discuss its drawbacks?

SAGE. My old friend, the glass can be half full as well, you know. [puffs and drinks wine]

SIMEONE. Your wine glass is almost empty. If you keep drinking at this rate the discussion will become impossible.

SALVA. Or more funny. [smiles] Back to CVA, boys. CVA looks like an option on the residual value of the traded portfolio of netting sets, with a random maturity given by the default time of the counterparty. Why an option, you may ask? To answer this question you need to look at cash flows. If the counterparty defaults before the final maturity and the present value of the portfolio at default is positive, then the surviving party only gets a recovery. If, however, the present value is negative, the surviving party has to pay it in full to the liquidators of the defaulted entity. Once we have done all calculations by netting these cash flows, we have that the value of the deal under counterparty risk is the value with no counterparty risk minus a positive adjustment, called CVA. This adjustment is the option price in the above sense. See for example Brigo and Masetti (2005) for details and a discussion. Of course, an option with random maturity is a complicated object. . . [looks at Sage and Simeone, sighs] It is complicated because it generates model dependence even in products that were model independent without counterparty risk.

SAGE. Let's use examples, so we avoid getting lost in the technobabble. [orders another glass of wine]

SIMEONE. Let us take for example a portfolio of plain vanilla swaps.

SALVA. [checks watch, adjusts skirt] You have to price an option on the residual value of the vanilla swap portfolio at default of the counterparty. You need an interest rate option model and, by adding counterparty risk, your valuation has become model dependent. Quick fixes to pricing libraries are quite difficult to obtain. And model dependence of

course means that volatilities and “correlations” would impact this calculation and similarly for dynamics features more generally.

SIMEONE. And then my problems start. It is very hard to measure implied volatilities and implied correlations, i.e. under \mathbb{Q} rather than \mathbb{P} .

SAGE. You do what you must, take some shortcuts, proxying, etc.

SALVA. But in any case no, they are not easy to measure. We are pricing and we are under \mathbb{Q} . We need volatilities and correlations extracted from traded prices of products that depend on such parameters. But from what market products do I imply information on the correlation between a specific corporate counterparty default and the underlying of the trade, for example gold, or a specific FX rate? And where do I extract credit spread volatilities from? Single name CDS options are not liquid.

SIMEONE. I agree, but it’s actually worse than that. For some small/medium and sometimes even large counterparties it is even difficult to imply default probabilities, not to mention expected recoveries. From what I understand [looks at Salva] \mathbb{Q} default probabilities can be deduced from credit default swap or corporate bond counterparty data. But how many small and medium enterprises have no reliable CDS or Bond quotes? Many counterparties do not have a liquid CDS or even an issued bond that are traded. What if the counterparty is the port of Mouston? Where can one imply default probabilities from, leave alone credit volatilities and credit-underlying “correlations”? Not to mention the often overlooked recoveries (and do not get me started on forty percent).

SAGE. Puffff. . . you are making a fuss for nothing. Default probabilities, when not available under \mathbb{Q} as you would say, may be considered under \mathbb{P} . You may then adjust them for an aggregate estimate of credit risk premia obtained from credit index data. For example, rating information can yield rough aggregate default probabilities for the port of Mouston if we have a rating system for small medium enterprises (these are available by credit agencies). One may also consider \mathbb{P} -statistics like historical credit spread volatility and historical correlation between underlying portfolio and credit spread of the counterparty. That allows you to model even wrong-way risk to some extent.

SALVA. [amused/sarcastic] I have to admit I am fascinated by the phenomenal speed you show in solving all problems. They really should consider you for the Nobel award in economics. [leaning forward, murmuring] Do you realize that in your answer above there are a number of very fundamental problems? Do you trust rating agencies? Do you think their ratings for all SME⁴ are reliable? Do you think most financial institutions have suitable internal rating systems? Do you think you can hedge a price you obtained through historically estimated statistics easily? Do you. . .

⁴small and medium-sized enterprises.

SAGE. Wait, isn't your Girsanov theorem saying that instantaneous vols and correlations for diffusions are the same under the two measures?

SALVA. [looking at Sage carefully] As usual, you pretend to be a jerk but you are quite smart. Yes, you would be right. But it's not that simple. Parameterizing the Radon-Nikodym...

SAGE. [finishes drinking the second glass] Please, Salva, stop. I told you, this is all about action. You should not be paralyzed by too much analysis and too many questions. [orders a third glass]

SALVA. This starts looking like "shoot first and ask questions later". Do you realize we are just scratching the surface of all the problems CVA has?

SIMEONE. I agree again. Are we having an honest debate or are we here to pretend everything is easy and doable and get drunk?

SAGE. You two are ganging up against me. It's not fair two against one. [feigns helplessness] And I am under the spell of this wonderful lady who has completely enchanted me.

SALVA. [ironic] Indeed, poor boy, look how we cornered you into a fatal position. You have been outsmarted by the femme fatale quant and by the Machiavellian Byzantine regulator. Checkmate. [laughs]

SAGE. Ahahaha, you are fantastic, we need to get out more often. [drinks more wine] All right [puffs], I will concede that the data may be a problem. But not an impossible one. And let me paraphrase Jules Verne: I am ignorant, sure, but I am so ignorant that I even ignore difficulties. They do not stop me. [winks]

SIMEONE. You should work as a comedian, not as a trader. Can we be serious for a minute? Let us talk about wrong-way risk. There is a lot of attention to that in the regulators space and we are not doing well there. Basel is having a particularly hard time with wrong-way risk.

SAGE. I don't see all this fuss about wrong-way risk. This is simply the additional risk we have when the underlying portfolio and the default of the counterparty are "correlated" in the worst possible way. Suppose, for example, that you are trading an equity forward with a corporate and you will receive fixed and paying (variable) floating equity at maturity. Suppose the equity of the forward and the equity of the counterparty to be positively correlated, for example the counterparty is Nokia and the underlying is Vodafone. We may have a negative correlation between the default likelihood of Nokia and Vodafone's

equity, since higher prices of Vodafone will lead to higher values of Nokia's equity, which in turn implies lower default likelihood for Nokia. On the opposite side, lower values of the Vodafone equity will lead to higher default probabilities for Nokia. When Vodafone's equity decreases importantly, the default probability of Nokia will increase a lot due to wrong-way correlation and the value of the residual receiver equity forward will increase as well. This means that the embedded CVA option term will be more in the money precisely in those situations where the default probability of the counterparty Nokia is larger, thus causing much more damage than in the case with low correlation. This is an example of wrong-way risk. The opposite case, with negative correlation between Nokia and Vodafone equities would be right-way risk.

SIMEONE. That is a good example.

SALVA. A good one, indeed. More generally, Wrong Way Risk has been studied, for example, in the following references in different asset classes: Redon (2006), Brigo and Masetti (2005), Brigo, Morini, and Tarengi (2011), Brigo and Tarengi (2004), Brigo and Tarengi (2005) for equity, Brigo and Pallavicini (2007), Brigo and Pallavicini (2008) for interest rates, Brigo, Chourdakis, and Bakkar (2008) for commodities (Oil), Brigo and Chourdakis (2008) for Credit (CDS).

SAGE. To our resident librarian! [bows, raises wine chalice, drinks] On the other hand, who needs to read all those papers? It's just as I told you.

SALVA. [smiling] "Also sprach Zarathustra"

SIMEONE. Look...

SALVA. Ok ok. I think another quite difficult problem is how to correctly frame the debt valuation adjustment, DVA. It has to do with the possibility that both parties in a deal agree on the counterparty risk charge. Maybe Sage, who is so good with examples, could start with some introduction?

SAGE. [kills cigar on the tray] I will tell you what I understood. [sips wine] Let's say that we are default-free and all recognize this. We are pricing the risk that the counterparty defaults before the final maturity of the deal on a given netting set. This is the CVA to us and as we have seen it is a positive adjustment to be subtracted from the default-risk-free price to us. Again, as we said above, having the choice and all things being equal, we prefer trading with a default risk-free counterparty rather than with a risky one. So we understand the risk-free price to us needs to be diminished by subtracting a positive quantity called CVA, so as to compensate us for the additional default risk. Now ask yourself: what happens from the point of view of the other party?

SIMEONE. You mean the other party looks at our default probability in the deal?

SAGE. No, or at least not yet. I mean the same setup as above, where we are still default free and the counterparty retains the default probability, but now it is the counterparty doing the analysis, not us.

SALVA. [looking at Sage admiringly] Very smart.

SAGE. The counterparty will mark a corresponding positive adjustment (the opposite of our negative one) to the risk-free price to her. This way both parties will agree on the price, because we agree we have to pay less for the contract (the price to us has decreased by CVA) and the counterparty agrees it has to pay more (the price to it has increased by the same amount, CVA). This makes sense. The adjustment for the counterparty client is positive because the counterparty needs to compensate us for its default risk. So finally we have gotten to DVA: the adjustment seen from the point of view of the counterparty is positive and is called debt valuation adjustment, DVA. It is positive because the early default of the counterparty herself would imply a discount on its payment obligations towards us, and this means a gain for her. So the counterparty marks a positive adjustment over the risk-free price by adding the positive amount called DVA. In this case, where we are default free, the DVA computed by the counterparty is also called Unilateral DVA, UDVA, since only the default risk of the counterparty is included. Similarly, the adjustment marked by us by subtraction is called Unilateral CVA, UCVA. In this case $UCVA(us) = UDVA(counterparty)$, i.e. the adjustment to the risk-free price is the same, but it is added by the counterparty and subtracted by us. Notice also that, since we are default-free, $UDVA(us) = UCVA(counterparty) = 0$.

SIMEONE. Very clear. But this is not yet a bilateral case really, since there is only the counterparty that may default. So this DVA is not the general one.

SAGE. You have the general one when the two firms do not agree on one of them being default free. Say that in your example the counterparty does not accept that we are default free (a reasonable objection, after the eight credit events that happened to Financial Institutions in one month of 2008).

SALVA. [finishing the first glass of wine] Allow me to step in. In this case then the only possibility for both parties to agree on a price is for both parties to consistently include their defaults into the valuation. Now both parties will mark a positive (bilateral) CVA to be subtracted and a positive (bilateral) DVA to be added to the default risk-free price (MtM) of the deal. The CVA of one party will be the DVA of the other one and viceversa. Both parties will compute the final price as follows:

$$MtM - CVA + DVA$$

and, in Sage's example,

$$Price\ To\ Us = MtM\ to\ Us + DVA(us) - CVA(us),$$

whereas when the counterparty does the calculation she gets a completely analogous formula

$$\text{Price To Counterparty} = \text{MtM to Counterparty} + \text{DVA}(\text{Counterparty}) - \text{CVA}(\text{Counterparty})$$

and, recalling that

$$\text{MtM to Us} = - \text{MtM to Counterparty},$$

$$\text{DVA}(\text{us}) = \text{CVA}(\text{Counterparty}), \quad \text{DVA}(\text{Counterparty}) = \text{CVA}(\text{us}),$$

we get that eventually

$$\text{Price To Us} = - \text{Price To Counterparty},$$

so that both parties agree on the price. We could call Total (bilateral) Valuation Adjustment (TVA) to one party the difference CVA - DVA as seen from that party,

$$\text{TVA} = \text{CVA} - \text{DVA}.$$

Clearly TVA to us = - TVA to counterparty. We need to pay attention to terminology. By “bilateral CVA” the market refers both to TVA and to the CVA component of TVA. Mostly the industry uses the term to denote TVA and we will do so similarly, except when explicitly stated otherwise.⁵

SIMEONE. What is the technical literature on Bilateral CVA and on DVA?

SAGE. Yaaawnn

SALVA. [ignoring Sage] The first calculations are probably Duffie and Huang (1996), but Wrong Way Risk is hard to model in their framework. Furthermore, that paper deals mostly with swaps. Again, swaps with bilateral default risk are dealt with in Bielecki and Rutkowski (2001), but the paper where bilateral risk is examined in detail and DVA derived is Brigo and Capponi (2008a). In that paper bilateral risk is introduced in general and then analyzed for CDS. In Brigo, Pallavicini, and Papatheodorou (2011), Brigo, Capponi, and Pallavicini (2014) and Brigo, Capponi, Pallavicini, and Papatheodorou (2011) other aspects of bilateral risk are carefully examined, also in relationship with wrong-way risk, collateral and extreme contagion and gap risk. Those works analyze what happens when default happens between margining dates and a relevant mark to market change for worse has occurred. Brigo, Capponi, and Pallavicini (2014) considers a case of an underlying CDS with strong default contagion where even frequent margining in collateralization is quite ineffective.

SIMEONE. All right, thank you Salva. So we can summarize this by saying that Total bilateral Valuation adjustment is the difference between CVA and DVA as seen by the party doing the calculation.

SALVA. Yes but be careful. TVA is not just the difference of CVA and DVA in a universe where only one name can default. In computing DVA and CVA in the difference, you

⁵As it is frequent in the counterparty risk literature, the same terminology is used by different authors and users with different meaning. So is the case with TVA.

need to account for both defaults in both terms. There is thus a first-to-default check: If we are doing the calculation, in scenarios where we default first the DVA term will be activated and the CVA term vanishes, whereas, in scenarios where the counterparty defaults first, our DVA vanishes and our CVA payoff activates. So we need to check who defaults first. However, some practitioners implemented a version of TVA that ignores first to default times. If we compute

$$\text{TVA(us)} = \text{CVA(us)} - \text{DVA(us)}$$

(see for example Picoult (2005)) we are computing DVA(us) in a world where only we may default and then compute CVA(us) in a world where only the counterparty may default. But we do not eliminate the other term as soon as there is a first default. So in a sense we are double counting. The appropriate TVA contains a first to default check. The difference between the two approximations has been considered in Brigo, Buescu, and Morini (2012). The error in neglecting the first to default term can be quite sizeable even in seemingly harmless examples.

SIMEONE. Understood. From the conversations I had with our regulated entities, it looks like the industry prefers to leave the first to default out because this avoids the need to model default correlation between the parties involved in the deal.

SAGE. That is not very important, since if the underlying netting set is credit sensitive (for example contains CDS) then you need to model default correlation anyway or you have no wrong-way risk.

SIMEONE. But I understand the large majority of deals are interest rate swaps, so for those with the simplified formula without first to default you could indeed use your Unilateral CVA libraries to compute the bilateral adjustment without bothering with default correlation.

SAGE. Sure. But I still don't understand all this fuss everyone is making about DVA.

SIMEONE. Well, it's not hard to see why. DVA is a reduction on my debt due to the fact that I may default, thus not paying all my debt and a missed debt payment is like a gain, but I only can realize this gain as a cash flow if I default. Do you find that natural or straightforward?

SAGE. Yes, it is quite natural actually.

SIMEONE. There is more: if your credit quality worsens and you recompute your DVA, you mark a gain.

SAGE. Which, again, is perfectly natural. What is wrong? Everyone coming up with this story.

SIMEONE. You are kidding, right? Let us look at some numbers. Citigroup, in its press release on the first quarter revenues of 2009, reported:

“Revenues also included [...] a net 2.5\$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”.

More recently, from the Wall Street Journal:

october 18, 2011, 3:59 PM ET. Goldman Sachs Hedges Its Way to Less Volatile Earnings. “Goldman’s DVA gains in the third quarter totaled \$450 million, about \$300 million of which was recorded under its fixed-income, currency and commodities trading segment and another \$150 million recorded under equities trading. That amount is comparatively smaller than the \$1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported \$1.7 billion of DVA gains in its investment bank. Analysts estimated that Morgan Stanley will record \$1.5 billion of net DVA gains when it reports earnings on Wednesday”

So it is a sizeable effect. Now you think this is not relevant? Let me ask you a question then. How could DVA be hedged? Because, you will agree with me, a price that is not backed by a hedging strategy is hard to implement and will hardly prevail over time. To make DVA real, one should have a hedging strategy, i.e. one should sell protection on oneself, a very difficult feat, unless one buys back bonds that he had issued earlier. This may be hard to implement.

SAGE. Come on, proxy hedging is not hard. Most times DVA is hedged by proxying. Instead of selling protection on oneself, one sells protection on a number of names that are highly correlated to oneself.

SIMEONE. I heard of this. The WSJ article reported:

“[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials. A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldn’t say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained “a peer group”. Most would consider peers to Goldman to be other large banks with big investment-banking divisions, including Morgan Stanley, J.P. Morgan Chase, Bank of America, Citigroup and others. The performance of these companies’ bonds would be highly correlated to Goldman’s”.

Now let me object to this. Proxying can be misleading. It can approximately hedge the spread risk of DVA, assuming the spread correlation is strong, but not the jump to default risk. Morgan hedging DVA risk by selling protection on Lehman would not have been a good idea. In fact this can worsen systemic risk when you look at jump-to-default risk. If I sell protection on a firm that is correlated to me and then that firm not only has its credit quality worsen (which would hedge my DVA changes due to spread movements)

but actually defaults, then I have to make the protection payments and that could push me into default. Do you think this is an unrealistic scenario?

SAGE. Protection proxying is not traded on a single name, but on a basket of names. So there is a diversification.

SALVA. Again let me step in. There is a contradiction here. On one hand you want the names to be as much correlated as possible with yourself, or the hedge won't be effective. On the other hand, to avoid systemic risk, you hope to lower such correlations through diversification.

SAGE. Well I am not saying that diversification lowers all correlation, but only systemic risk correlation.

SALVA. And what other correlation is there? Are you thinking about a factor structure? I am sceptical. Think about the eight credit events that happened in one month of 2008 to Financials.

SIMEONE. That's right, my sentiments exactly.

SAGE. All right, I will concede again that it is probably not as simple as I think. I have not considered the big picture carefully, I need to think about it more.

SALVA. That would be a first. [smiles, orders a bottle of water, looks at Sage a little mysteriously] Now do not feel alone in this. Regulators are having a very hard time in deciding what to do with DVA. [leans back, adjusts skirt] Simeone, why don't you tell us what is happening with DVA in the regulatory space?

SIMEONE. [didactically] Basel III recognizes CVA risk but does not recognize DVA risk. This is creating a gap between CVA calculations for capital adequacy and CVA calculations for accounting and mark to market. For example (I can be a bit of librarian too):

“This CVA loss is calculated without taking into account any offsetting debt valuation adjustments which have been deducted from capital under paragraph 75”. (Basel III, page 37, July 2011 release)

“Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements”. (FAS 157)

SAGE. [rolls his shirt sleeves] I see that regulators have clear ideas.

SIMEONE. [ignoring Sage, pinching chin] And here is what the former president of the Basel Committee said:

“The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognize it, says Stefan Walter, secretary-general of the Basel Committee: The main reason for not recognizing DVA as an offset is that it would be inconsistent with the overarching supervisory prudence principle under which we do not give credit for increases in regulatory capital arising from a deterioration in the firm’s own credit quality”.

SALVA. See? You are not alone in your DVA perplexity. [crosses legs, adjusts skirt]

SIMEONE. [frowning, joining hands] Another problem where we are trying to obtain some clarity from ISDA is closeout.

SALVA. Yes, that is interesting. I worked on that a little, so let me summarize. [smiles at both] Closeout is what happens when the first of the two parties in the deal defaults. So suppose in our example the counterparty defaults. Closeout proceedings are then started according to regulations and ISDA⁶ documentation. The closeout procedure fixes the residual value of the contract to us and establishes how much of that is going to be paid to us. If, however, it is negative then we will have to pay the whole amount to the counterparty. So far this seems the standard definition we have seen above. But think carefully: at the default time of the counterparty, do we value the remaining contract by taking into account our own residual credit risk (namely by including our now unilateral DVA, “replacement closeout”) or just by using a default risk-free valuation (“risk-free closeout”)? The replacement closeout maintains that if we are now going to re-open the deal with a risk-free party, the risk-free party will charge us a unilateral CVA, which, seen from our point of view, is our unilateral DVA. Hence in computing the replacement value we should include our DVA to avoid discontinuity in valuation. If we always included DVA to value the deal prior to default, we should not stop doing so at default. So we should use replacement closeout. On the other hand, since we are liquidating the position now, why bother about residual credit risk? So we should use risk-free closeout.

SAGE. [looking at Salva admiringly] Interesting, does the valuation change strongly?

SALVA. In Brigo and Morini (2011), Brigo and Morini (2010a) and Brigo and Morini (2010b) it is shown that a risk-free closeout has implications that are very different from what we are used to expecting in case of a default in standardized markets such as the bond or loan markets. Let us take a case of TVA where the valuation is always in the same direction, e.g. a loan or a bond. Suppose we own the bond that we bought at time 0

⁶International Swaps and Derivatives Association.

from the counterparty. We are waiting for the payment of the final notional and we paid in the beginning, so we are like a lender in a loan. If we default, i.e. default of the lender, this should imply no losses to the bond issuer or to the borrower (the counterparty in our example). Instead, if the risk-free default closeout is adopted, if we default then the value of the liability of the counterparty will suddenly jump up. In fact, before the default, the liability of the counterparty (net debtor) had a mark-to-market that took into account the risk of default of the counterparty itself. In other words, it had a DVA term as seen from the counterparty. As a negative cash flow, it was smaller, in absolute value, than if DVA had not been there. After our default, if a risk-free closeout applies, this mark-to-market transforms into a risk-free one, surely larger in absolute value than the pre-default mark-to-market, because DVA is no longer there. The increase will be larger the larger the credit spread. This is a dramatic surprise for the counterparty that will soon have to pay this increased amount – no longer offset by DVA – to our liquidators. This effect does not exist in the bond or loan market. Clearly, net debtors at default (the counterparty in our example) will not like a risk-free closeout. They will prefer a replacement closeout, which does not imply a necessary increase of the liabilities since it continues taking into account the credit-worthiness of the debtor also after the default of the creditor.

SAGE. [whistles] I hadn't thought about this. But it does make sense...

SALVA. One could then decide to use replacement closeout. However, the replacement closeout has shortcomings opposite to those of the risk-free closeout. The risk-free closeout will be preferred by the creditors. The more money debtors pay, the higher the recovery will be. The replacement closeout, while protecting debtors, can in some situations worryingly penalize the creditors by abating the recovery. This for example happens when the defaulted entity is a company with high systemic impact, so that when it defaults the credit spreads of its counterparties are expected to jump high.

SAGE. It seems unbelievable that no clear regulation was available for this issue. What are you waiting for, Simeone??

SIMEONE. There are other issues, it's not over yet with the list of problems. Closeout is the least of our worries.

SALVA. So what are the other issues that are keeping you regulators busy?

SIMEONE. Collateral modeling, especially rehypothecation. Capital requirements on CVA for Basel III. Deciding what types of CVA restructuring we should allow. Consistent inclusion of funding costs. Proper conservative accounting of Gap Risk. Central Clearing. But I think we covered enough for today. I have to say that I enjoyed it more than I had thought. Same time tomorrow?

SALVA. [stands up, straightens skirt, picks up purse] Sure.

SAGE. You have been wonderful, Salva [kisses his cheeks]

SIMEONE. Salva, it must have been similar to kissing an ahstray [kisses Salva's cheeks in turn, shakes hands with Sage]

SAGE. Ahahah very funny. See you tomorrow, guys.

1.4 The Third Day

1.4.1 FVA, Hard Maths with No Data? CVA VaR, Basel III Problems, Collateral and Gap Risk

SAGE. Hi Simeone, let's sit outside. This year we are very lucky with the weather.

SIMEONE. Hi, Sage. Indeed we have wonderful weather this year. There, Salva is joining us.

SALVA. Hi Boys. How are you today? Why, what is this bouquet of roses on the table? [smiling, amused]

SAGE. I don't know. It was there. Look, there is a tag.

SALVA. "To the beautiful charming and intelligent lady at the outside table, from a secret admirer". [bows head slightly, looks penetratingly at Sage and then at Simeone] Mmmmm. [smiles] Anyway, my turn to buy drinks. Perhaps I'll find a secret admirer inside the pub. What would you like? [...]

SALVA. So how did the day go?

SAGE. More and more meetings. I am spending days meeting people who are mostly bureaucrats. Not really exciting. I am itching for action.

SIMEONE. It's funny that despite being in quite different institutions, we end up doing very similar activities. I have been doing that too.

SALVA. That's three of us then. Today I beat the record of "useless meetings". [laughs musically] All right, gentlemen, let us restart from where we ended up yesterday. [pensively runs left hand along long hair] We covered a lot of issues in the first two days and I believe the brainstorming was quite helpful. [leans forward, counting on fingers] We tried

to clarify the difference between risk measures like Credit VaR and pricing quantities like CVA, we tried to explain \mathbb{P} and \mathbb{Q} , we tried to reason on CVA and DVA hedging, on the conflicting regulations on DVA and then we talked about closeout, analyzing a little the risk-free vs replication closeouts and we talked about first-to-default risk. We also commented on the difficulties of finding data for the calculation of CVA.

SIMEONE. We have an interesting picture emerging from this debate. CVA and DVA seem to be so full of choices. I heard this interesting comment at an industry panel recently: five banks may be computing CVA in 15 different ways. And it is no surprise: from all the choices you listed, one is not even sure about which payout should be implemented for CVA, let alone the models. So how can a bank price it while being sure that other banks price it consistently? This is an issue for regulation.

SAGE. [rolls shirt sleeves] It does not matter. I simply price it and charge the other desks. If they are not happy, we may discuss. When the market opens, they may be buying CVA protection somewhere else through CCDS or similar structures, and then it will be simply an offer and demand process.

SIMEONE. [pinching chin] What if bank B manages to sell protection to bank A, not because it is doing a better job than bank C, but because it is pricing CVA with a sloppy payout and thus is obtaining a lower price, but based on assumptions that do not reflect reality? We already commented on the article by Watt (2011), that implicitly seems to suggest this may happen.

SAGE. [waving hand] Look, if a bank is sloppy in its procedures, this is that bank's problem, and not a problem with CVA.

SIMEONE. [raising hand] But the problem stems precisely from doubts about the CVA definition and conventions. So it is not entirely the bank's responsibility.

SALVA. [leaning forward] Simeone, I think this is what we expect from regulators to some extent. ISDA in particular is working on this, although I must say I have not been impressed with their work in the CDS "Big Bang" story of Beumee, Brigo, Schiemert, and Stoye (2010). But let's not digress.

SIMEONE. ISDA is not a regulator, strictly speaking. It is basically a trade organization for the over-the-counter derivatives market participants. It is not something like the BIS,⁷ for example.

SALVA. You are right, I was using the term regulator in a loose sense. Anyway I think that regulation is quite tricky. Especially because it would require the best people, a little

⁷BIS stands for the Bank of International Settlements.

like politics and police work. However, compensation levels often drive the most energetic and talented professionals to the industry rather than regulators. Present company excepted, of course.

SAGE. [lights a cigar, laughs] Perhaps I can convince you to work for us, Simeone. If you find ways the bank can reduce capital requirements I think you will get very large bonuses and you can stop driving around that old Toyota. [winks]

SIMEONE. [folding arms] Toyota is a great car, it never let me down in fifteen years. Unlike your top cars, I hear.

SALVA. Ok, leave toys alone. Let us talk about funding costs modeling, which is another very controversial topic in the industry.

SIMEONE. That was the subject of today's "useless meeting", as you say.

SALVA. [raising a calming hand] Yes, that is a quite popular subject. If you attend a practitioner conference, a lot of talks will be on consistent inclusion of funding costs, mostly through very patched solutions. Only a few works try to build a consistent picture where funding costs are included together with the aspects we discussed earlier, like CVA, DVA, collateral, closeout, etc. For example, the working paper Crépey (2011), then published in Crépey (2012a) and Crépey (2012b), is a comprehensive treatment. The only limitation is that it does not allow for underlying credit instruments in the portfolio and has possible issues with FX, but this has now been fixed in Crépey and Song (2014). Crépey (2011, 2012a, 2012b, 2014) are technical papers based on a mathematical instrument called a backward stochastic differential equation (BSDE). A related framework that includes most recent literature as a special case is in Pallavicini, Perini, and Brigo (2011, 2012), who however does not resort to BSDEs explicitly and starts from different assumptions on the risk-free rate and the risk neutral measure. Another good example of related work studying the funding issue is Bielecki and Rutkowski (2013), where the issue is studied in the martingale pricing and hedging framework, also referring to BSDEs when needed. Earlier works are partial but still quite important.

SAGE. [puffs] Yaaawn, prof skip intro. Let's roll, like yesterday. [sips wine]

SIMEONE. [annoyed, unfolding arms, palms on the table] I am quite interested in the literature review, if you do not mind.

SALVA. [waving hand dismissively towards Sage] The industry paper Piterbarg (2010) considers the problem of replication of derivative transactions under collateralization but without default risk, and in a purely classical Black-Scholes framework, and then considers two relatively basic special cases. When you derive the first basic results that way, you have to be very careful about the way you formulate the self-financing condition.

That paper is very influential, but the self-financing condition was not well stated there. A similar problem regarding the self-financing condition happens in some other industry papers. Luckily, this does not seem to ruin the final result but when you write papers on funding you have to pay particular attention to the self-financing condition. This is just to give you an idea of the difficulties in this area. Even top quants cannot have everything 100% right.

SAGE. [puffs] Fascinating, but let us leave aside the sociology of the practitioner's industry awards, technical communications and interaction with academia. What is the point of collateral without default risk, as you mentioned in the reference above? What is collateral used for then?

SALVA. [smiling wryly, looks at Sage sideways] If you let me develop a discourse I can try to be more comprehensive. There is not only default risk in collateral, there is also liquidity risk, transaction costs... and you might still be modeling credit spread risk but not jump to default risk, a distinction that is natural in intensity models. This involves filtrations, by the way, but I won't scare you with such technicalities.

SAGE. [puffs] Oh please, scare me. I am at your mercy, my wonderful femme fatale. [drinks]

SALVA. [ignoring Sage] However, it is true that the main reason to have collateral is default risk. Otherwise one would not have collateral in the first place. And it is not easy to account for all such features together, which may explain the hand-waving of default risk in papers like Piterbarg (2010) and the claim that including default risk would be easy. The fact that it is not can be easily grasped by looking at Crépey (2011), then published in Crépey (2012a, 2012b), for example. Following Piterbarg (2010), the fundamental funding implications in presence of default risk have been considered, in simple settings, first in Morini and Prampolini (2011); see also Castagna (2011). These initial works focus on zero coupon bonds or loans. One important point in Morini and Prampolini (2011) is that in simple payoffs such as bonds or loans, DVA has to be interpreted as funding, in order to avoid double counting. However, this result is not general and does not extend to more general payoffs. In the general case different aspects interact in a more complex way and the general approach of Crépey (2011, 2012a, 2012b) or Pallavicini, Perini, and Brigo (2011, 2012) is needed. To complete a first basic literature review, the paper by Fujii and Takahashi (2011a) analyzes implications of currency risk for collateral modeling, while the above-mentioned, Burgard and Kjaer (2011b) resorts to a PDE approach to funding costs. As I mentioned above, Crépey (2011, 2012a, 2012b) or Pallavicini, Perini, and Brigo (2011, 2012) and Bielecki and Rutkowski (2013) remain the most general treatments of funding costs to date. Then...

SAGE. [resting cigar on ashtray rim, raising his glass] Ok, I don't care who is interested in this, I have had enough of references and bla bla bla. Can you summarize the funding

problem difficulties? To me, funding seems to reduce to discounting, with different curves depending on the party we are dealing with and I don't get why it should be such a formidable problem. Explain please, but in plain English. Or I will simply look at you, which is a pleasant activity, but without listening to what you are saying.

SALVA. [crossing legs, adjusting skirt, concealing irritation behind a wry smile] All right, I'll humor you and pretend to believe you are not interested in modeling details. In a nutshell, when one needs to manage a trading position, one has to obtain funds in order to hedge the position, post collateral, pay coupons and swaps resets and so on. These funds may be obtained from one's treasury department or in the market. One may also receive funds, not from the treasury, but as a consequence of being in the position: a coupon, a notional reimbursement, a positive mark-to-market move, getting some collateral back, a closeout payment, etc. All flows need to be remunerated: if one is borrowing funds, this will have a cost and if one is lending funds, this will provide revenues in form of interest. Including funding costs into valuation means properly accounting for such features.

SAGE. [sipping wine] Which, as I said, can be done through discounting with different curves. So what is difficult here?

SALVA. [sighing] The point is doing this consistent with all other aspects, especially with counterparty risk. I think you heard of this "funding valuation adjustment", or FVA, that would be additive, so that the total price of the netting set portfolio would be something like

$$\text{MtM} - \text{DVA} - \text{CVA} - \text{FVA} = \text{MtM} - \text{TVA}.$$

However, this is too much to expect. Proper inclusion of funding costs leads to an implicit (or recursive if you want, as in a fixed-point equation) pricing problem. This may be formulated as a backward stochastic differential equation (BSDE, as in Crépey (2011,2012a, 2012b) or Bielecki and Rutkowski (2013)), or to a discrete time backward induction equation or nonlinear PDE (as in Pallavicini, Perini, and Brigo (2011, 2012)). Moreover, there is some overlap (or double counting) between DVA and FVA. One possible way out of double counting, used in Crépey (2012b), is to replace the final formula FVA term above by LVA, a term for the funding liquidity valuation adjustment (and there is also a last term, dubbed RC, for replacement cost, reflecting the mismatch between the value of the contract right before the default and its valuation by the liquidator in the closeout procedure). This is a possible choice but is not the only one. For example, Pallavicini, Perini, and Brigo (2012) renounce separating the adjustment's different quantities because they are deemed to be intrinsically non-separable. [sees Sage stirring, raises a flat hand]

I am getting there, be patient. I saw you flash your eyes when I mentioned recursion. I'm lucky you don't have heat vision. Why is the problem inherently recursive? The value of the cash and collateral processes depends on the price and on the future cash flows of the derivative which, in turn, depends on the choices for such processes, transforming the pricing equation into a recursive (and nonlinear) equation! Thus, funding and investing

costs cannot be considered as a simple additive term to a price obtained by disregarding them. Importantly, identifying DVA with funding is wrong in general, except that in the simple cases in Morini and Prampolini (2011), but see also the discussion in Castagna (2011).

SAGE. [taking cigar] And you said above that this is done well with BSDEs?

SIMEONE. Indeed. I was wondering about that too. Salva, you know we are not as advanced as you are mathematically. Could you give us a summary on what BSDEs are?

SALVA. [smiling, looks at Sage sideways, then looks at Simeone] Since you are asking so nicely. [sips wine] You know that a pricing problem is usually represented either as an expected value or as an (integro) partial differential equation for the price. . .

SAGE. The two representations being connected by the Feynman-Kac theorem, we know.

SALVA. . . . for simple pricing problems, the pricing can be expressed like a straight expectation, in which case we could roughly say that the pricing problem is linear. This is not the case for American put options, for example, where early exercise introduces a nonlinearity.

SIMEONE. Indeed, I remember this from my master's days.

SALVA. However, if you have a kind of recursive relationship similar to what we get with the funding problem, or if you have both early exercise and path dependency, you need to combine backward induction with forward simulation. If the size of the portfolio is very large, as is typical with CVA, you will be suffering with the curse of dimensionality if you use PDEs. The alternative is to use BSDEs. These equations need a terminal condition and then allow one to implement a sort of backward induction similar to what you see in least squared Monte Carlo for early exercise options. Actually, Brigo and Pallavicini (2007) first applied this idea to CVA, but in a case where the pricing problem is still linear. This idea, now called American Monte Carlo, is at the centre of the CVA system built by UBS and promoted in the book by Cesari, Aquilina, Charpillon, Filipovic, Lee, and Manda (2010). The general idea, implemented more generally through BSDEs, goes like this: basically, at every backwards induction step, on the forward simulated trajectories of the underlying assets, one uses a multilinear regression to estimate the value of the deal one time step earlier based on the underlying at the next time and on the price scenarios at the next time, which is known since we are going back in time. Because this is based on simulation and regression, it is also suited to high-dimensional situations, at least more so than the PDE approach. And if the problem is nonlinear one cannot trivially translate the PDE formulation into an expectation using the classical Feynman-Kac theorem, so that the use of BSDEs becomes relevant. On these technical issues, in relation to CVA, I recommend the recent book by Crépey (2013).

SAGE. This may sound convincing to a quant audience, but I doubt the banks will be willing to implement BSDEs and I also doubt the regulators will prescribe that. We need something simple coming out of this.

SIMEONE. As a regulator I think this last comment of yours is very important. It raises the question: if we realize that a project requires mathematical or methodological tools that are too advanced for the times, will we give up the related project and activities or should we keep it going being content with using completely inadequate methodologies?

SAGE. [puffs] I hope you are not suggesting that I should stop my business because I don't want to implement BSDEs. Do you realize how ridiculous that is?

SIMEONE. It's annoying for you, maybe, but not ridiculous. What if the pharmaceutical companies reasoned like you?

SAGE. Who says they don't? [laughs]

SALVA. Look, I am not saying you necessarily need to implement a BSDE, but definitely you need to start thinking along those lines and seriously consider the above recursion.

SIMEONE. I still think that, with uncertainty on the recoveries and occasionally on probabilities of default, volatilities and correlations that are very large, insisting on a fully rigorous and precise mathematical treatment of second order effects could be pointless. Is this right?

SALVA. I don't think so. The fact that you can't estimate recovery precisely does not mean you should give up properly understanding the remaining aspects. And in fact a mistake in the implementation of funding can cause quite large differences. Of course, if you are risk measuring CVA, then maybe you can accept being less precise than in pricing. However I think that Basel III is really simplifying too much.

SIMEONE. CVA VaR seems to mix everything together. Could you summarize CVA VaR for me?

SALVA. Let us say that Credit VaR measures the risk of losses you face due to the possible default of some counterparties you are doing business with. CVA measures the pricing component of this risk, i.e. the adjustment to the price of a product due to this risk. Fine. But now suppose that you revalue and mark to market CVA in time. Suppose that CVA moves in time and moves against you, so that you have to book negative losses, NOT because the counterparty actually defaults, but because the pricing of this risk has changed for the worse for you. So in this sense you are being affected by CVA volatility

and you are booking CVA mark to market losses. To quote Basel III, even if I still haven't found out how they computed this 2/3 number:

“Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty risk were due to CVA losses and only about one-third were due to actual defaults.”

In other words, the variability of the price of this risk over time has done more damage than the risk itself. This is why Basel is considering setting up quite severe capital charges against CVA.

SIMEONE. That is a good summary. [looks at Salva approvingly] To measure the risk in CVA mark to market, we have to compute VaR for CVA. Can you summarize how this would be computed?

SALVA. You simulate basic market variables and risk factors under \mathbb{P} , up to the risk horizon. Then, in each scenario at the horizon, you price the residual CVA until the final maturity, using a \mathbb{Q} expectation (but in a full approach this may be a recursion, as we commented above). You put all the prices at the horizon time together in a histogram and obtain a profit and loss distribution for CVA at the risk horizon. On this \mathbb{P} distribution you select a quantile at the chosen confidence level. Now you will have computed VaR of CVA. If you take the tail expectation of this histogram, you have expected shortfall of CVA. I stress again that this does not measure default risk directly. It measures the risk to have a mark to market loss due either to default or to adverse CVA change in value over time.

SAGE. [puffs] Let's get to the juicy part. What is Basel III prescribing for this CVA VaR?

SALVA. Well the framework has changed several times: bond equivalent formula, multipliers, tables... One of the main issues has to do with (WWR). In some part of the Basel regulation it had been argued that you could calculate CVA as if there were no wrong-way risk and then use a standard multiplier/table/adjuster to account for wrong-way risk. In other words, you should assume independence between default of the counterparty and of the underlying portfolio, compute CVA and then multiply by a certain number to account for correlation risk, or something along those lines. However, this does not work. Depending on the specific dynamics of the underlying financial variables, on volatilities and correlations and on the chosen models, the hypothetical multipliers are very volatile. See again Brigo, Morini, and Tarenghi (2011), Brigo and Pallavicini (2007), Brigo, Chourdakis, and Bakkar (2008) and Brigo and Chourdakis (2008) for examples from several asset classes. Even if one were to use this idea only for setting capital requirements of well diversified portfolios, this could lead to bitter surprises in situations of systemic risk. [sits back, uncrosses and stretches legs, finishes glass of wine, orders a soft drink]

SAGE. Looks like a very intensive numerical procedure. Pricing CVA in each risk hori-

zon scenario would require a simulation, so you have subpaths. And, if the portfolio PV is not known in closed form, even the residual NPV at default inside the CVA term requires a simulation, so you would need three levels of simulations, impossible even for current technology. Suppose you need $10000 = 10^4$ scenarios at every step. Then the total number of paths would be $10000^3 = 10^{12}$, one trillion scenarios. So it is clear you can't do that by brute force (yet).

SALVA. Very perceptive. [smiles]

SIMEONE. So the only way out of capital depleting CVA VaR charges is collateral?

SALVA. [straightening skirt] Well, as I said earlier collateral does not completely kill CVA, this is folklore. Gap risk is real. Gap risk shows up when default happens between margining dates, and a relevant negative mark to market move happens very quickly. Brigo, Capponi, and Pallavicini (2014) show a case of an underlying CDS with strong default contagion where even frequent margining in collateralization is quite ineffective. But it is recognized that, in cases without strong default contagion, collateral can be quite effective.

SAGE. [drops cigar on ashtray] You know, this has been very educational but more tiring than I thought. Could we stop here for today and perhaps conclude tomorrow?

SALVA. Fine by me. Today I am the one who did most of the talking so I would stop gladly. [smiles]

SIMEONE. I am tired as well.

[usual exchange of handshakes and kisses]

1.5 The Fourth Day

1.5.1 Counterparty Risk Restructuring. CCDS, Papillon, Floating Rate CVA and Margin Lending. Global Calibration. Global Valuation. Available CVA Books and Forthcoming CVA Books.

SIMEONE. My first question today is this. From what we said yesterday, Basel III may end up imposing heavy capital requirements for CVA. Collateralization is a possible way out, but it may become expensive for some firms and it may lead to a liquidity strain, while firms that are not organized for posting collateral will not be able to resort to collateral in the first place. The article by Watt (2011) reports the case of Lufthansa:

“The airline’s Cologne-based head of finance, Roland Kern, expects its earnings to become more volatile “not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions.”

For banks, the choice is whether to collateralize everything or to be subject to heavy CVA capital requirements. Is there a third way? I read about some attempts to restructure counterparty risk differently.

SAGE. I can tell you. . . There have been proposals for instruments that can hedge CVA, or reduce its capital requirements in principle. Contingent credit default swap (CCDS) anyone?

SIMEONE. CCDS, yes, recently standardized by ISDA after a long time.

SALVA. Yes I remember CCDS. . . a CCDS is similar to a CDS, but when the reference credit defaults, the protection seller pays protection on a notional that is not fixed but is rather given by the factor $(1 - \text{recovery})$ fraction of the residual value of the Portfolio netting set at that time, if positive. This matches exactly the CVA on the given portfolio if the reference credit of the CCDS is the counterparty of a deal. Back in 2008 the Financial Times commented:

“[. . .] Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope. [. . .] Counterparty risk has become a particular concern in the markets for interest rate, currency and commodity swaps - because these trades are not always backed by collateral. [. . .] Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions (Financial Times, April 10, 2008).”

Interest in CCDS came back in 2011-2012 as CVA capital charges were becoming punitive. ISDA has launched CCDS on index portfolios, i.e. standardized portfolios that are thought to be representative of the typical netting sets components banks are facing. This is an attempt to increase standardization. However, CCDS do not fully solve the problem of CVA capital requirements. First of all, there is no default-free bank from which the CCDS protection can be bought, so that CCDS are themselves subject to counterparty risk. Second, I doubt the ISDA standard portfolios will match any realistic bank portfolio. So there will be only a partial hedge that ignores cross-asset classes correlation and especially the structure of realistic netting sets. Better than nothing, but very partial.

SAGE. Yeah, and this is prompting the industry to look for other solutions that may also be effective across several counterparties at the same time and on complex netting sets.

SIMEONE. CVA securitization could be considered, although the word “securitization” is not very popular these days.

SALVA. Is there any proposed form of CVA restructuring, or securitization?

SAGE. [concentrating, looking tired] I need a drink, but probably a coffee. There are a few structures. I am familiar with a few deals that I cannot disclose. However, there have also been deals discussed in the press and in the Financial Times Alphaville blog, in particular Pollack (2012a):

“In short, Barclays has taken a pool of loans and securitised **TO: securitized** them, but retained all but the riskiest piece. On that riskiest Euro 300m, Barclays has bought protection from an outside investor, e.g. hedge fund. That investor will get paid coupons over time for their trouble, but will also be hit with any losses on the loans, up to the total amount of their investment. To ensure that the investor can actually absorb these losses, collateral is posted with Barclays.”

SALVA. Looks like a CDO?

SAGE. Yes, and collateral is key here. The blog continues:

“This point about collateral means that, at least in theory, Barclays is not exposed to the counterparty risk of the hedge fund. This is especially important because the hedge fund is outside the normal sphere of regulation, i.e. they aren’t required to hold capital against risk-weighted assets in the way banks are. [...] And then there is the over-engineering element whereby some deals were and maybe still are, done where the premiums paid over time to the hedge fund are actually equal to or above the expected loss of the transaction. That the Fed and Basel Committee were concerned enough to issue guidance on this is noteworthy. It’ll be down to individual national regulators to prevent “over-engineering” and some regulators are more hands-on than others.”

SIMEONE. Wasn’t another deal, called “Score”, discussed as well?

SAGE. Yes, but it didn’t work well, Pollack (2012c):

“RBS had a good go at securitising these exposures, but the deal didn’t quite make it over the line. However, Euroweek reports that banks are still looking into it:

“Royal Bank of Scotland’s securitisation of counterparty risk, dubbed Score 2011, was pulled earlier this year, but other banks are said to be undeterred by the difficulties of the asset class and are still looking at the market. However, other hedging options for counterparty risk may have dulled the economics of securitising this risk since the end of last year”.”

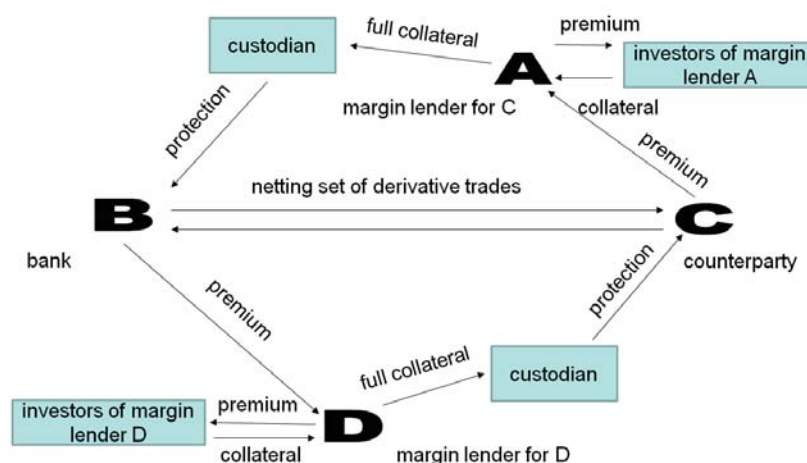


FIGURE 1.1: General counterparty scheme, including quadri-partite structure.

SIMEONE. The most effective one was probably the Credit Suisse one, Pollack (2012b):

“Last week Credit Suisse announced it had bought protection on the senior slice of its unusual employee compensation plan. The Swiss bank pays some of its senior bankers using a bond referencing counterparty risk, which also involves shifting some counterparty risk from the bank to its workers”.

That is like buying protection from your own employees. Interesting concept if you think about it from a regulator’s point of view. That way the employee, in theory, is incentivized in improving the risk profile of the company.

SAGE. [getting mad] You are kidding, right? I wouldn’t be too happy if I were paid that kind of bonus. My decision doesn’t have the capacity to affect the large scale performances of the group. That is more for the CEO and company. So let the CEO bonus be like that, but not my own.

SIMEONE. For your benefit, there are actually more innovative ideas. On CVA securitization see for example Albanese, Bellaj, Gimonet, and Pietronero (2011), that advocates a global valuation model. The more model-agnostic Albanese, Brigo, and Oertel (2013) explains how margin lending through quadri-partite or penta-partite structures involving clearing houses would be effective in establishing a third way. Let me borrow from Albanese, Bellaj, Gimonet, and Pietronero (2011) and Albanese, Brigo, and Oertel (2013), to which I refer for the full details. If I understood correctly, the structure is like [Fig. 1.1].

Traditionally, CVA is charged by the structuring bank *B* either on an upfront basis or it is built into the structure as a fixed coupon stream. This other Score deal we discussed

above (there is also Papillon) is probably of this type too. Margin lending instead is based on the notion of floating rate CVA payments with periodic resets.

SALVA. Let me guess what Floating rate CVA is: assume for simplicity a bi-partite transaction between the default-free bank B and the defaultable counterparty C. Instead of charging CVA upfront at time 0 for the whole maturity of the portfolio, the bank may require a CVA payment at time 0 for protection on the exposure up to 6 months. Then in 6 months the bank will require a CVA payment for protection for further six months on what will be the exposure up to one year and on and on, up to the final maturity of the deal. Such a CVA would be an example of Floating Rate CVA.

SAGE. Yeah, makes sense.

SIMEONE. Good. I was saying that margin lending is based on the notion of floating rate CVA payments with periodic resets, and is designed in such a way so as to transfer the conditional credit spread volatility risk and the mark-to-market volatility risk from the bank to the counterparties. We may explain this more in detail by following the arrows in Fig. 1.1. The counterparty C has problems with posting collateral periodically in order to trade derivatives with bank B. To avoid posting collateral, C enters into a margin lending transaction. C pays periodically a floating CVA to the margin lender A, which the margin lender A pays to investors. This latest payment can have a seniority structure similar to that of a cash CDO. [raises hand] Wait. In exchange for this premium, for six months the investors provide the margin lender A with daily collateral posting and A passes the collateral to a custodian. This way, if C defaults within the semi-annual period, the collateral is paid to B to provide protection and the loss is taken by the investors who provided the collateral. At the end of the six months period, the margin lender may decide whether to continue with the deal or to back off. With this mechanism C is bearing the CVA volatility risk, whereas B is not exposed to CVA volatility risk, which is the opposite of what happens with traditional upfront CVA charges.

SAGE. The idea is interesting, but I can see a number of problems. First, proper valuation and hedging of this to the investors who are providing collateral to the lender is going to be tough. I recall there is no satisfactory standard for even simple synthetic CDO tranches. One would need an improved methodology.

SALVA. Actually, I heard that recently a paper on dynamic copulas and simultaneous defaults has been proposed that solves the CDO bottom-up / top-down dilemma by displaying both a realistic aggregated loss dynamics (good calibration of CDO tranches) and single names information. See Bielecki, Cousin, Crépey, and Herbertsson (2013a), who generalize earlier works such as Brigo, Pallavicini, and Torresetti (2007) (see also the summary in Brigo, Pallavicini, and Torresetti (2010)).

SAGE. But still... I am sure it remains a formidable problem. And the other problem

that, I am sure, has not escaped Simeone is: what if all margin lenders pull out at some point due to a systemic crisis?

SIMEONE. That would be a problem. Albanese, Brigo, and Oertel (2013) submit that the market is less likely to arrive in such a situation in the first place if the wrong incentives to defaulting firms are stopped and an opposite structure, such as the one in Fig. 1.1, is implemented. I would also point out that this structure probably also helps with the funding implications.

SALVA. We haven't talked about CCPs and initial margins. You know, often people object that all these CVA and funding issues will disappear once a CCP is intermediating the deal.

SAGE. Well, as long as you are happy with the CCP knowing the details of your transactions, and you're also happy to pay the possibly daily initial margins, besides variation margins.

SIMEONE. Initial margins are supposed to protect you from wrong-way risk and gap risk. So they should be a good thing.

SAGE. Except that they are not passed to the client but are kept by the CCP itself. So what would become of your initial margin if the CCP defaulted? It happened in the past.

SALVA. You see, to analyze whether initial margins are fair or appropriate, you need exactly the type of credit and funding analytics that have been developed recently in papers like Crépey (2011,2012a, 2012b) and Pallavicini, Perini, and Brigo (2011, 2012).

SAGE. I think we have reached the border of this new land. We need to celebrate. Hey maestro, Prosecco please!

SIMEONE. I would ask Salva for suggestions for further reading while we wait for the bottle.

SAGE. You are really a spoilsport. Who cares about books now?

SALVA. [laughs] Why don't you drop the recital? You read more books than I do, the only difference is that you do it during the night (and I don't know how you manage such an energy level).

SAGE. Oh, by now I mostly meet people, shake hands and send email. I am a big boss now. [laughs]

SIMEONE. All right, all right. It should only take five minutes?

SALVA. Ok, no worries. So let us list the counterparty risk / CVA books.

- The first book we can mention is Pykhtin (2005). This is a collection of papers on CVA and is quite interesting, although it is now a little outdated.
- The book by Cesari, Aquilina, Charpillon, Filipovic, Lee, and Manda (2010) is more recent. From a modeling point of view it is rather basic but it also looks at the IT implications of building a CVA system. Relying on the so called American (or least square) Monte Carlo technique introduced for CVA computations in Brigo and Pallavicini (2007, 2008), it addresses a number of practical problems that deal with CVA for realistically large portfolios.
- The book by Gregory (2009, 2012) is technically quite simple but explains basic CVA concepts in a clear way and is good especially for managers and finance people who need to get a general grasp of CVA fundamentals with some elements about funding/discounting, without going too technical. It is quite popular and has been successful.
- The book by Kenyon and Stamm (2012), although technically basic, is original in that it tackles current and relevant problems such as multi-curve modeling and credit valuation adjustments, with closeout and especially goodwill, which cannot be found anywhere else to the best of my knowledge. Funding costs, hints at systemic risk, regulation and Basel III are also considered.
- The book by Brigo, Morini, and Pallavicini (2013) is mostly based on Brigo's work with several co-authors during the period 2002-2012 and has been written to be widely accessible while being technically advanced. It has a very readable introductory part written in the form of a Platonic dialogue. The book deals with complex issues such as CVA and DVA, with several examples of advanced wrong-way risk calculations across asset classes. It also shows how to model gap risk, collateral in general, closeout, rehypothecation and funding costs. The funding part is based on Pallavicini, Perini, and Brigo (2011, 2012). Several numerical studies illustrate the fine structure of wrong-way risk. The final part of the book provides cutting edge research on CVA restructuring through CCDS, CDO tranches type structures, floating rate CVA and margin lending.
- The book by Crépey, Bielecki, and Brigo (2014) is probably the most advanced. It also has a very readable introductory part in the form of a long and entertaining Galileian dialogue. . . wait, I feel a strange sensation of being sucked into a different reality. . . into the book actually . . . strange . . . OK, I was saying that this is an advanced book dealing with CVA, DVA and funding, collateral and wrong-way risk. The funding part is based on Crépey (2011, 2012a, 2012b) and is at the forefront of current research, similar to Pallavicini, Perini, and Brigo (2011, 2012) in terms of generality. By comparison with the book by Brigo, Morini, and Pallavicini (2013),

the emphasis is on the mathematical dependence structure of the problem, using mainstream stochastic analysis: BSDEs in particular, in line with Crépey (2013), to address the nonlinear recursive nature of the funding issue in a more systematic way, and dynamic copulas to reconcile bottom up and top down perspectives in credit models. These are the two puzzles referred to in the title, which make the book quite unique. By the way there is a possible connection between the two: dynamic copula models can also be analyzed in terms of BSDEs, as the final chapter illustrates (see also Crépey and Song (2014)). The book also contains a number of numerical studies.

These are the books I heard of, but there are probably others. But did you feel that strange pull I felt while speaking about Crépey, Bielecki, and Brigo (2014)?

SAGE. Actually yes, I felt a strange situation of being watched and then being embedded in a recursion, I cannot explain it...

SIMEONE. Oh you explained it even too well, I felt the same!!

SAGE. Oh, here comes the Prosecco. Great, let's drink so we can forget these strange feelings.

SIMEONE. But the label on the bottle says Taylor and Francis?? [thunder]

SAGE. What? [sky becomes suddenly dark]

SALVA. [eyes wide open, long hair in the wind] Oh my, you are right. Don't give me that. I am not part of a meta-narrative experiment. I have had enough of Grant Morrison, Jostein Gaarder and Jasper Fforde, and even of the final part of Brigo, Morini, and Pallavicini (2013). I am real, leave me alone.

SAGE. What? This can't be true...