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RESEARCH STATEMENT

My research interests are in probability, and more precisely, they include superprocesses, interacting particle systems, random trees and percolation. In my PhD work, I established limit theorems for super-Brownian motion and for the voter model. More recently, in a joint work with Omer Angel and Jesse Goodman, we have studied the scaling limit of invasion percolation on a regular tree. Also, in collaboration with Iddo Ben-Ari and Alexander Roitershtein, we studied a one-dimensional random walk, whose drift is function of the number of visits at the origin. After introducing these objects (Section 1), I will describe the results I obtained (Section 2) and future research plans (Section 3).

1. Introduction

Both particle systems and percolation have been studied extensively in probability theory since the late fifties. They were first introduced to describe models in statistical mechanics. Since then, these objects have been as well thought of as being models for epidemics (oriented percolation and the contact process), behavioral systems, models for competing species (the voter model), neural networks, genetic evolution. Although part of the motivation came from physics or biology, mathematicians have studied these models for their own sake.

1.1. Interacting particle systems.

A particle system typically consists of a finite or infinite set of particles in which each particle evolves in a finite or countable state space E. Without interaction, the particles would evolve as independent continuous time Markov chains with state space E. The "interaction" usually refers to the fact that the motion of the particle in E depends on the state of neighboring particles.

A first example is the critical branching particle system, which can be seen as a model for an evolving population, where one keeps track not only of the genealogy of individuals but also of their location in space. It is described in the following way. Particles (or individuals) move independently in space according to a given Markov process W, and, at rate 1 independent exponential times die and give birth to a random number of offspring according to a given distribution with mean 1 and positive finite variance. In the case when W is d-dimensional random walk, this system is called *branching random walk* and it was proven by Watanabe that it possesses a scaling limit, later called "super-Brownian motion" by Dynkin.

The *voter model* is a second example of interacting particle system. It was initially introduced as a model for two competing species* denoted 0 and 1. At any time $t \geq 0$, each site $x \in \mathbb{Z}^d$ is occupied by an individual which belongs to one of the two species. Such an individual has a rate 1 exponentially distributed lifetime. When it dies, it is immediately replaced by a new individual, whose species is chosen among those of its neighbors, according to the jump kernel p on $\mathbb{Z}^d \times \mathbb{Z}^d$. The jump kernel p is fixed, and we assume it is irreducible, centered, symmetric, translation invariant, isotropic, and that it

*it is also possible to interpret 0 and 1 as two opinions hold by voters placed at each vertex of the lattice, which is the reason for the name given to this model has exponential moments†. Lifetimes and neighbor choices are assumed independent. The state of this model at time t is described by the set of positions of individuals belonging to specie 1, denoted $\xi_t \subset \mathbb{Z}^d$, or by the measure $\sum_{x\in\xi_t}\delta_x$.

Recently, Cox, Durrett and Perkins have etablished that in dimension $d \geq 2$, a conveniently rescaled version of the voter model converges towards super-Brownian motion. We use this convergence to estimate hitting probabilities of a far point for the voter model in $d \geq 2$ (see 2.2).

It should be emphasized that the study of the voter model is made easier by the fact that its dual is a system of coalescing random walks.

The *Lotka-Volterra model* can be described as a modified version of the voter model. The replacement mechanism remains identical, but here, lifetimes of the individuals depend on local densities f_0, f_1 of individuals of type 0 and 1. More precisely, for given parameters $\alpha_0 > 0, \alpha_1 > 0$, an individual of type 1 at x dies at rate $f_1(x,\xi) + \alpha_1 f_0(x,\xi)$, and similarly, an individual of type 0 at x dies at rate $f_0(y,\xi) + \alpha_0 f_1(x,\xi)$. The case $(\alpha_0,\alpha_1) = (1,1)$ exactly corresponds to the voter model. We further notice that for $\alpha_i < 1$, an individual of the species i survives longer when surrounded by individuals of type $1-i$. On the contrary, for $\alpha_i > 1$, species i fares better in the presence of individuals of its own type. This model is much harder to study than the voter model. However, Cox, Durrett and Perkins have recently proven that in $d \geq 2$, a rescaled version (where in particular, one lets a sequence of parameters (α_0^N, α_1^N) go to $(1,1)$ at the right speed) of Lotka-Volterra models converges to a super-Brownian motion with drift. This result allowed Cox and Perkins to study Lotaka-Volterra model, close to $(1,1)$, and in dimension $d \geq 3$. The case $d = 2$ is more delicate (see section 3).

1.2. **Invasion percolation.** Bond percolation models a fluid flow in a random medium. We consider an infinite graph G, whose edges $e, e \in G$ are assigned i.i.d uniform random variables X_e on $(0,1)$. For a given parameter $p \in [0,1]$, the edge (or bond) e is said to be occupied (open) if $X_e \geq p$, while it is said to be vacant (closed) when $X_e < p$. It is well-known that the existence of an infinite cluster of closed bonds undergoes a phase transition, as there is a critical value $p_c \in [0,1]$ such that for $p < p_c$ there is no infinite cluster with probability 1, while for $p > p_c$ there exists an infinite cluster with probability 1.

Invasion percolation is a related stochastic growth model, which is a nice example of self-criticality. An infinite subgraph of G is grown inductively as follows. Define I_0 to be the vertex o. For $N \geq N_0$, given I_N, let I_{N+1} be obtained by adjoining to I_N the edge in its boundary with smallest weight X_e. The invasion percolation cluster is the random infinite subgraph $\bigcup_{N\geq N_0} I_N \subset G$.

Invasion percolation and critical percolation are closely related. Suppose indeed that $\mathcal{A}$ is a critical percolation cluster in G. If the edge $I_{N+1} \setminus I_N$ gets into $\mathcal{A}$, then one will invade every vertex in $\mathcal{A}$ before being able to leave it. When building the IPC, one thus explores successively different critical percolation clusters, and one can expect that larger and larger clusters are invaded. This intuition was confirmed by Chayes, Chayes and Newmann who proved that for any $\varepsilon > 0$ the number of edges in the IPC with weight greater than $p_c + \varepsilon$ is almost surely finite. Therefore, it is natural to compare IPC to IIC (incipient infinite cluster, defined by Kesten). In this direction, Jarai established that in $\mathbb{Z}^2$, IIC and IPC measures were equivalent far from the root. One could be tempted to believe then that scaling limits of the two objects, if they exist, would be the same.

Recent work by Angel, Goodman, den Hollander and Slade have shown otherwise. They obtain a structural representation of the IPC *on a regular tree*, which in particular allows one to understand that IPC and IIC are "locally" equivalent far from the origin, but *globally* different. One can conjecture that such a phenomenon also takes place in $\mathbb{Z}^2$.

† for example, one can think of the nearest-neighbor jump kernel

We use this structural representation to find the scaling limit of IPC on a regular tree (see paragraph 2.3). We also describe the distinct limit of the IIC on this regular tree. Finally, the random walk model introduced in 2.4 is partially inspired by this problem.

We should note here that a particular case of percolation is linked to super-Brownian motion. *Oriented percolation* corresponds to the case when the oriented edges of the graph of $\mathbb{Z}^d \times \mathbb{N}$ are those $\{(i,n),(j,n+1)\}$, for i and j neighbors in $\mathbb{Z}^d$ and $n \in \mathbb{N}$. Van der Hofstadt and Slade proved recently that scaling limit of oriented percolation can be expressed in terms of the canonical measure of super-Brownian motion, for $d > 4$. Furthermore, van der Hofstadt, den Hollander and Slade have established that scaling limit of the IIC of oriented percolation can be expressed in terms of excursion measure of super-Brownian motion conditioned to survive forever, for $d > 4$.

1.3. **Super-Brownian motion, random trees.** In fact, super-Brownian motion has recently appeared at the limit of a wider range of interacting particle systems, including for instance lattice trees and contact process. It seems that super-Brownian motion can be thought of, in a similar way to Brownian motion, as a universal object, which provides information on a variety of discrete models.

Numerous path properties of super-Brownian motion (such as hitting probabilities, Hausdorff measure properties of the support and of the range), as well as its links with a class of partial differential equations were established in the eighties, by Dawson, Dynkin, Perkins and others. Part of my PhD thesis has consisted in establishing new asymptotic results for the occupation measure and additive functionals of super-Brownian motion (see paragraph 2.1)

Moreover, this precise knowledge of the path properties of super-Brownian motion proves extremely useful for establishing asymptotic properties of the discrete models introduced in 1.1 (see also paragraphs 2.1, 2.2 and 3).
We should note that a number of the properties of super-Brownian motion were in fact studied in the larger class of superprocesses, for which the spatial displacement is a general Markov process, and the branching mechanism is not anymore necessarily quadratic.

The Brownian snake approach to super-Brownian motion (and in particular its canonical measure, or "excursion measure"), due to Le Gall, allows to clearly separate the spatial displacement from the quadratic branching mechanism. It also allows a very simple description of the integrated super-Brownian excursion appearing in the limit of unoriented percolation. It further extends to the description of a general superprocess.

This approach was inspired by the work of Aldous, who gave sense to the scaling limit of rescaled Galton-Watson trees conditioned to survive. This scaling limit is a continuum tree which can be expressed in terms of a normalized Brownian excursion. Recent work of Duquesne and Le Gall allowed to extend these results to the case of a more general class of branching mechanisms.

We make use of the theory of Duquesne and Le Gall in our study of the scaling limit of the IPC introduced in 1.2 (see paragraph 2.3).

2. Description of the results

2.1. **Asymptotic results for the occupation measure and additive functionals of super-Brownian motion.**

2.1.1. *Local behavior of local times of super-Brownian motion* [1]. Let X be a super-Brownian motion under the probability measure $\mathbb{P}_{X_0}$. For a function $f : \mathbb{R}^d \to \mathbb{R}$, the notation $\langle X_t, f\rangle$ is shorthand for $\int_{\mathbb{R}^d} f(x)X_t(dx)$. For $d \le 3$, there exists a random continuous function $(t,x) \to L^x_t$ from $(0,\infty)\times\mathbb{R}^d$ into $\mathbb{R}_+$ such that for any bounded continuous function Ψ on $\mathbb{R}^d$,

$$\int_0^t < X_s, \Psi > ds = \int_{\mathbb{R}^d} \Psi(x) L^x_t dx. \tag{1}$$

L_t^x is called the *local time* of X at point $x \in \mathbb{R}^d$ and time $t > 0$. Local times of superprocesses have been studied by many authors (such as Sugitani, Adler and Lewin, Krone). In [1], I obtained precise asymptotics for the local times of super-Brownian motion. For simplicity, I will describe here a special case of the main theorem.

Let $x_0 \in \mathbb{R}^d \setminus \{0\}$ and consider the process Xdefined under the measure $\mathbb{P}_{\delta_{x_0}}$. Set $\psi(c) = \sqrt{c}$ if $d = 3$, $\psi(c) = c/\ln(c)$ if $d = 2$. Then $(\psi(c)(L_t^{x/c} - L_t^0))_{t\geq 0}$ converges, as $c \to \infty$, to $(\beta^{(x)}_{a(x)L_t^0})_{t\geq 0}$ where β is a one-dimensional Brownian motion *independent* of X (and $a(x)$ an explicit constant depending only on x).

In fact, the result is more general, it is valid for any initial measure whose closed support does not contain the origin, and the statement involves finitely many values of x (say $x_1, ...x_k$) and the precise correlations between β^{x_i} and β^{x_j} for $1 \leq i < j \leq k$.

Let K be a compact set of $\mathbb{R}^d$. As can be guessed from formula (1), this result allows to obtain limit theorems for the occupation measure of a compact set K/c for a super-Brownian motion started at δ_{x_0}. More precisely, let ϕ and ξ denote K-supported integrable functions such that $\int_K \phi(x)dx \neq 0$, $\int_K \xi(x)dx = 0$. Set ϕ_c, ξ_c to be the functions defined by the relations $\phi_c(x) = \phi(cx)$, $\xi_c(x) = \xi(cx)$. Then for a super-Brownian motion X started from μ, and $t > 0$,

$$\left(c^d \int_0^t < X_s, \phi_c > ds, c^d\psi(c) \int_0^t < X_s, \xi_c > ds\right) \underset{c\to\infty}{\overset{\text{(law)}}{\longrightarrow}} \left(L_t^0 \int_K \phi(x)dx, U_t\right), \tag{2}$$

where, conditionally on L_t^0, U_t is Gaussian with variance $a_\xi L_t^0$, and a_ξ is a constant depending only on ξ.

In $d = 3$, after rescaling, this allows to recover and extend an analytic result of Lee, who obtained a limit theorem for the occupation measure of K for a super-Brownian motion started at δ_{cx_0}, and conditioned to hit K.

2.1.2. *Limit theorems for the occupation measure of super-Brownian motion,* [2]. This joint work with J-F. Le Gall was motivated by the following simple question. Consider the population of a branching random walk system (with branching coefficient γ), introduced in 1.1. Suppose that initially, this population reduced to a single individual located at a point x far from the origin, and condition on the event that an individual will eventually visit the origin. Then, what is the typical number of individuals who visit the origin?
We address a continuous version of this problem, and consider a d-dimensional super-Brownian motion X with branching rate γ, starting at a distant site x. The order of the probability for X to hit the unit ball B_1 is known from results of Dawson, Iscoe and Perkins (89). For a super-Brownian motion conditioned to hit B_1, we study the asymptotics of the distribution of mass over B_1. More precisely, if $\mathcal{Z}$ denotes the total occupation measure of super-Brownian motion, and φ is a nonnegative measurable function on $\mathbb{R}^d$, we are looking at the asymptotic behavior of the variable $f_d(x)\langle \mathcal{Z}, \varphi\rangle$ under $\mathbb{P}_{\delta_x}(\cdot \mid \mathcal{Z}(B_1) > 0)$, as $|x| \to \infty$, and where f_d is a convenient renormalization.

In the case $d \geq 5$, we prove that for $f_d = 1$, the law of this variable converges to a probability distribution whose moments can be expressed, thanks to known recursive formulas for the moments of $\mathcal{Z}$ under the excursion measure $\mathbb{N}_x$ of super-Brownian motion. Furthermore, in the case $d \leq 3$, $f_d = |x|^{d-4}$ and the convergence directly follows from the existence of a continuous local time, and in fact, the limit (2) discussed in the previous paragraph provides a more precise result.

The most interesting part concerns the critical dimension $d = 4$, in which case we establish that the law of $(\ln(|x|)^{-1}\langle \mathcal{Z}, \varphi\rangle$ under $\mathbb{P}_{\delta_x}(\cdot \mid \mathcal{Z}(B_1) > 0)$ converges as $|x| \to \infty$ to an exponential distribution with mean $\gamma(4\pi^2)^{-1}\int \varphi(y)dy$.

3. Hitting probability of a distant point for the voter model [3]

Consider the voter model introduced in 1.1, and suppose that initially there is only one individual of type 1, located at the origin of $\mathbb{Z}^d$. We denote by $(\xi_t^0, t \geq 0)$ the process, and P the probability measure under which it is defined. Recall that ξ_t^0 is the set of points $x \in \mathbb{Z}^d$ such that the individual at x and at time t belongs to the species 1. We also recall that the transition kernel $p : \mathbb{Z}^d \times \mathbb{Z}^d \to [0,1]$ is symmetric, translation invariant, centered, isotropic and with exponential moments. The isotropy assumption means there exists $\sigma \in (0, \infty)$ such that

$$\forall i, j \in \{1, ..d\}, \quad \sum_{y \in \mathbb{Z}^d} y^i y^j p(0, y) = \sigma^2 \delta_{ij}.$$

Let $x \in \mathbb{R}^d \setminus \{0\}$, $c \in \mathbb{R}_+^*$, and let $[x]_c$ be the point in $\mathbb{Z}^d/c$ closest to x with some convention when there is more than one such point. In [3], the main goal of my work is to obtain asymptotics on $P(\exists t \geq 0 : c[x]_c \in \xi_t^0)$. Let us define

$$\phi_d(c) = \begin{cases} \frac{c^2}{\ln(c)} & \text{if } d = 2, \\ c^2 & \text{if } d = 3, \\ c^2 \ln(c) & \text{if } d = 4, \\ c^{d-2} & \text{if } d \geq 5. \end{cases}$$

let $\beta_2 = 2\pi, \beta_3$ be the probability that a rate 1 random walk in $\mathbb{Z}^3$ with jump kernel p started at the origin never returns to the origin. Then, if $d = 2$ or $d = 3$,

$$\lim_{c \to \infty} \phi_d(c) P(\exists t \geq 0 : c[x]_c \in \xi_t^0) = \frac{2\sigma^2}{\beta_d} \left(2 - \frac{d}{2}\right) |x|^{-2}. \tag{3}$$

If $d \geq 5$, there exist positive constants a_d, b_d depending on x such that

$$a_d \leq \liminf_{c \to \infty} \phi_d(c) P(\exists t \geq 0 : c[x]_c \in \xi_t^0) \leq \limsup_{c \to \infty} \phi_d(c) P(\exists t \geq 0 : c[x]_c \in \xi_t^0) \leq b_d.$$

In dimension 4, I obtain less precise results. For some positive a_4,

$$\liminf_{c \to \infty} \phi_4(c) P(\exists t \geq 0 : c[x]_c \in \xi_t^0) \geq a_4.$$

However, the existence of a positive b_4 such that a statement similar to the ones in dimension $d \geq 5$ holds is only conjectured. Nevertheless, the same proof as in $d = 2$ or 3 leads to

$$\limsup_{c \to \infty} c^2 P(\exists t \geq 0 : c[x]_c \in \xi_t^0) = 0.$$

The proof of this result in dimensions 2 and 3 is particularly interesting because it uses the corresponding results for occupation measure of super-Brownian motion and the fact, proved earlier by Bramson, Cox and Le Gall, that the scaling limit of the voter model started with a single one and conditioned to survive is super-Brownian motion under its excursion measure $\mathbb{N}_0$, with branching coefficient $2\beta_d$ and diffusion constant σ^2.

Moreover, the proof of (3) requires interesting intermediate results. One of these expresses that the probability for the voter model under $P_\alpha^* := P(.|\xi_\alpha \neq \emptyset)$ to escape $B(0, A)$ decays exponentially with A. One also shows that under P_α^*, the range of the voter model does not contain any "isolated" point, with probability arbitrarily close to 1 when α is taken large enough.

3.1. **Scaling limit of the invasion percolation cluster on a regular tree, in collaboration with Omer Angel and Jesse Goodman** [5]. For $\sigma \in \mathbb{Z}_+^* \setminus \{1\}$, we consider a regular tree $\mathcal{T}_\sigma$ (in which any vertex has degree $\sigma + 1$, except for the root o which has degree σ). We let (IPC) denote the invasion percolation cluster on $\mathcal{T}_\sigma$ as defined in 1.2, and (IIC) the incipient infinite cluster on $\mathcal{T}_\sigma$.

The structural representation of the (IPC) due to Angel, Goodman, den Hollander and Slade (2006) is the following. The (IPC) is a discrete tree which possesses a single infinite rising backbone, uniformly distributed among all rising paths from the root to infinity. From this backbone, denoted BB, emerge, at every height and in every direction away from it, subcritical percolation clusters. The values of the parameters of subcritical percolation clusters are function of the height at which they emerge (more precisely they depend on the maximal weight of edges of the backbone above this height). When climbing up the backbone, these values $(\hat{W}_k, k \geq 0)$ converge to the parameter of critical percolation $1/\sigma$. Moreover, for any $\epsilon > 0$,

$$(k(1-\sigma \hat{W}_{[kt]}), t \geq \epsilon) \underset{k\to\infty}{\overset{\text{(law)}}{\longrightarrow}} (L(t), t \geq \epsilon). \tag{4}$$

The process L above is determined by its graph, which is the lower envelope of a Poisson point process on $(0,\infty)\times(0,\infty)$ with intensity 1.

As mentioned in paragraph 1.2, this description allows to explain the similarities and differences between (IPC) and (IIC). The latter is indeed also a discrete tree with a (uniform) single infinite rising backbone, from which emerge *critical* percolation clusters. It turns out that the convergence of the subcritical parameters $\hat{W}_k$ towards the critical value $1/\sigma$ is slow enough so that the scaling limits of these two objects is different.

One may code a sin-tree (a tree with a single infinite rising backbone) $\mathbf{T}$ with a pair of *height functions*. The first, denoted $H_G^{\mathbf{T}}$, values at time n the height (the generation) of the nth vertex — for the lexicographical order — of $\mathbf{T}$. The second, denoted $H_D^{\mathbf{T}}$, codes in a similar way the mirror image $\mathbf{T}^\bullet$ of $\mathbf{T}$. We define these two coding functions as functions of $\mathbb{R}_+$, by linearly interpolating between two consecutive integers. In [5], we prove the convergence of rescaled versions of the pairs of coding functions for both (IPC) and (IIC). For a given process X we let $\underline{X}_t := \inf\{X_s, s \leq t\}$. With respect to the topology of uniform convergence on compact intervals,

$$\begin{aligned}&\left(\frac{1}{k}H_G^{(\text{IPC})}(k^2t), \frac{1}{k}H_D^{(\text{IPC})}(k^2t), t\geq 0\right)\\ &\quad\underset{k\to\infty}{\overset{\text{(law)}}{\longrightarrow}} \left(\sqrt{\frac{4\sigma}{\sigma-1}}(Y_t - 2\underline{Y}_t), \sqrt{\frac{4\sigma}{\sigma-1}}(\tilde{Y}_t - 2\underline{\tilde{Y}}_t), t \geq 0\right).\end{aligned}$$

In the expression above,

$$Y_t = B_t - \int_0^t L\left(-\underline{Y}_s\right) ds, \quad \tilde{Y}_t = \tilde{B}_t - \int_0^t L\left(-\underline{\tilde{Y}}_s\right) ds,$$

B and $\tilde{B}$ are two independent standard Brownian motions and L is the lower enveloppe process appearing in (4).
The case of the (IIC) is easier, and corresponds informally to replace L by 0 in the above discussion. We obtain

$$\begin{aligned}&\left(\frac{1}{k}H_G^{(\text{IIC})}(k^2t), \frac{1}{k}H_D^{(\text{IIC})}(k^2t), t\geq 0\right)\\ &\quad\underset{k\to\infty}{\overset{\text{(law)}}{\longrightarrow}} \left(\sqrt{\frac{4\sigma}{\sigma-1}}(B_t - 2\underline{B}_t), \sqrt{\frac{4\sigma}{\sigma-1}}(\tilde{B}_t - 2\underline{\tilde{B}}_t), t \geq 0\right).\end{aligned}$$

Here again, B, $\tilde{B}$ are two independent standard Brownian motions, (alternatively speaking, $B - 2\underline{B}$, $\tilde{B} - 2\underline{\tilde{B}}$ are two independent Bessel processes of dimension 3).

We also describe the scaling limits of other processes coding (IIC), (IPC), as well as those of the trees themselves. Finally, this convergence allows us to describe the law of the scaling limits of the volume and surfaces of balls in the (IPC) in terms of the limiting height functions. In the latter case, this leads to a simple expression of the scaling limit of the surface of balls of the (IPC).

3.2. **A random walk on $\mathbb{Z}$ whose drift is function of the number of visits to the origin, in collaboration with Iddo Ben-Ari and Alexander Roitershtein** [4]**.** We fix a sequence of positive reals $(\varepsilon_n)_{n\in\mathbb{N}}$ which goes to 0 at infinity. We consider a nearest-neighbor random walk X on $\mathbb{Z}$, with a drift at every point but the origin. This drift always points towards 0, and its value at time n is ε_{η_n}, where $\eta_n = \{p : 0 \leq p \leq n, X_p = 0\}$. The main difficulty here is that X is not a Markov chain (however (X, η) is).

In order to relate this model with the preceding discussion, we note that X is none other that the coding (Lukaciewicz path) of a binary sin-tree with a right backbone, and whose subtree emerging at height n on the backbone, if non-empty, can be identified with a binary Galton-Watson tree with subcritical branching parameter $1 - \varepsilon_n$.

We let P be the probability measure under which this walk is defined, and $\mathbb{P}$ the law of the simple random walk on $\mathbb{Z}$.

When the sequence ε converges sufficiently fast towards 0, it is not surprising that the asymptotic behavior of this walk is not different from that of the simple random walk. More precisely, we prove that when $n\varepsilon_n \to 0$, the functional central limit theorem remains valid, moreover the measures P and $\mathbb{P}$ are mutually absolutely continuous if and only if $\sum \varepsilon_n < \infty$.

When the convergence of ε towards 0 is slower, our walk behaves very differently. Suppose $n\varepsilon_n \to \infty$ and that ε is regularly varying. Then, the position of the walk, when suitably rescaled, converges to a symmetric exponential variable (moreover, the excursion straddling n also possesses a non-degenerate scaling limit. Furthermore, in this case, position and maximum of the walk scale differently. The intermediate regime is more delicate (see paragraph 3.3 below for the case when $n\varepsilon_n \to$ cste).

4. Projects

4.1. **Coexistence for the two-dimensional Lotka-Volterra model, project with Ed Perkins.** Cox and Perkins have established coexistence results (i.e. the existence of an invariant measure which charges configurations with infinitely many 0's and infinitely many 1's) for the Lotka-Volterra model in $d \geq 3$. Coexistence was proved for values $\alpha_0 < 1, \alpha_1 < 1$ sufficiently close to $(1, 1)$. Their approach is based on a comparison with the dynamics of the voter model and the result of the convergence of rescaled Lotka-Volterra model towards a super-Brownian motion with drift.

We conjecture that this result remains valid in the two-dimensional case (which is most relevant for biological applications). But the two-dimensional situation is much more delicate. For instance, coexistence does not hold for the 2-dimensional voter model whereas it holds for $d \geq 3$. Moreover, the first order drift now vanishes for $\alpha_0 = \alpha_1 < 1$. The approach is based on proving a new limit theorem for a sequence of Lotka-Volterra models. This is done through looking at the martingale problems satisfied by the sequence of rescaled models. The convergence of most of the terms in these martingale problems can be dealt with similarly as in Cox and Perkins. There is however a new drift term, which requires some careful analysis, and in particular, requires some new estimates on coalescing random walks.

4.2. **Sin-trees.** One would be tempted to generalize the result obtained for (IPC) and (IIC). Indeed, it is possible, for $\beta > 1$, to obtain the process $B_t - \beta \underline{B}_t$ as a limit of rescaled height functions of well-chosen sin-trees. Such a process only is a diffusion for $\beta = 0, 1$ or 2. The study of these processes is therefore complicated when $\beta \in \mathbb{R}_+^* \setminus \{1, 2\}$. The approach in terms of trees would allow in particular a more simple computation of the law of the occupation measure of intervals, as well as that of the local time at a given height. One could also investigate an extension to the more general case when the Brownian motion B is replaced with a drifted one, or a stable process, or even a general Lévy process. One can also consider the case of processes such as the one discussed in the next paragraph.

4.3. **The critical case of the walk of paragraph 2.4, in preparation with Alexander Roitershtein.** When $n\varepsilon_n \to 1$, we are able to study the walk described in paragraph 2.4. We obtain that it possesses a scaling limit, unique in law solution of the equation

$$Y_t = B_t - \int_0^t \frac{\mathrm{sgn}(Y_s)}{\ell_s^0(Y)} ds.$$

Above, $\ell_s^0(Y)$ is the local time of Y at the origin, and at time s. We may define the process $(Y, \ell^0(Y))$ through its excursion measure from $\{0\} \times [0, \infty)$, and the proof of the convergence relies on a decomposition of the trajectory of (Y, η) into its excursions away from $\{0\} \times \mathbb{N}$.

References

[1] Merle M., Local behavior of local times of super-Brownian motion *Ann. Inst. Henri Poincar.*42 (2006), pp 491-520

[2] Le Gall J.-F., Merle M., On the occupation measure of super-Brownian motion, *Electronic Comm. Probab.*11 (2006), pp. 252-265.

[3] Merle M., Hitting probabilities of a distant point for the voter model, accepted for publication in *Ann. Probab.*, `http//www.imstat.org/aop/future_papers.htm`.

Submitted :

[4] Ben-Ari I., Merle M., Roitershtein A. A random walk on $\mathbb{Z}$ with drift driven by its occuaption time at zero, soumis à *Stoch. Proc. Appl.*, `http//arxiv.org/abs/0711.4871`

Manuscript :

[5] Angel O., Goodman J., Merle M., The scaling limit of the invasion percolation cluster on a regular tree, manuscript, `http//www.math.ubc.ca/~merle`