IMPORTANT ERRATUM

The proof of Lemma 4.1 in [1] is completely false, because the small function $\delta(\eta)$ actually depends on p in (4.6). This issue is not easily fixed but fortunately, this can be done using a theorem of Tasković-Alonso-Gamba-Pavlović [2]. Lemma 4.1-(i) was used only to prove Corollary 2.3, point (iii), concerning existence of a weak solution with some bounded exponential moments. Lemma 4.1-(ii) was not used.

Tasković-Alonso-Gamba-Pavlović prove *in particular* in [2, Theorem 2.4-(ii)], see also (2.17) for the link between Mittag-Leffler moments and classical exponential moments, the following result. Assume that the collision kernel is of the form

$$B(|v-v_*|,\theta)\sin^{d-2}\theta = |v-v_*|^\gamma \beta(\theta) \tag{1}$$

for some $\gamma \in (0,1]$ and some measurable $\beta : (0,\pi] \to \mathbb{R}_+$ such that

$$\int_0^\pi \theta^2 \beta(\theta) \mathrm{d}\theta < \infty. \tag{2}$$

Then for any $K > 0$, $\epsilon_0 > 0$, there are $C > 0$ and $\epsilon_1 > 0$ such that for any weak solution to the homogeneous Boltzmann equation preserving kinetic energy,

$$\int_{\mathbb{R}^3} e^{\epsilon_0 |v|^\gamma} f_0(\mathrm{d}v) \le K \quad \Longrightarrow \quad \sup_{t\ge 0} \int_{\mathbb{R}^3} e^{\epsilon_1 |v|^\gamma} f_t(\mathrm{d}v) \le C.$$

Thanks to this result of [2], Corollary 2.3-(iii) in [1] holds true under the assumptions $\Phi(z) = z^\gamma$ (i.e. (1)) and (**A2**), i.e. $\int_0^\pi \theta\beta(\theta)\mathrm{d}\theta < \infty$ (which is stronger than (2)).

True hard potentials do satisfy these conditions.

References

[1] N. Fournier, C. Mouhot, On the well-posedness of the spatially homogeneous Boltzmann equation with a moderate angular singularity, *Comm. Math. Phys.* **289** (2009), 803–824.

[2] M. Tasković, R.J. Alonso, I.M. Gamba, N. Pavlović, On Mittag-Leffler moments for the Boltzmann equation for hard potentials without cutoff, *SIAM J. Math. Anal.* **50** (2018), 834–869.