ERRATUM

Gytis Kulaitis pointed out some typoes in the statement of Theorem 15 of [1]. The proof seems correct.

Recall we consider a $\mathbb{R}^d$-valued Markov chain $(X_n)_{n\geq 1}$ with transition kernel P and initial distribution $\nu \in \mathcal{P}(\mathbb{R}^d)$ and we set $\mu_N := N^{-1}\sum_1^N \delta_{X_n}$. We assume that it admits a unique invariant probability measure π and the following L^2-decay property (usually related to a Poincaré inequality)

$$\forall\, n \geq 1,\ \forall\, f \in L^2(\pi), \quad \|P^n f - \pi(f)\|_{L^2(\pi)} \leq \rho_n \|f - \pi(f)\|_{L^2(\pi)} \tag{1}$$

for some sequence $\rho = (\rho_n)_{n\geq 1}$ decreasing to 0.

Theorem 15. *Let $p \geq 1$, $d \geq 1$ and $r > 2$ be fixed. Assume that our Markov chain $(X_n)_{n\geq 0}$ satisfies (1) with a sequence $(\rho_n)_{n\geq 1}$ satisfying $\sum_{n\geq 1}\rho_n < \infty$. Assume also that the initial distribution ν is absolutely continuous with respect to π and satisfies $\|d\nu/d\pi\|_{L^r(\pi)} < \infty$. Assume finally that $M_q(\pi) < \infty$ for some $q > pr/(r-1)$. Setting $q_r := q(r-1)/r$ and $d_r = d(r+1)/r$, there is a constant C, depending only on $p, d, r, q, \rho, M_q(\pi)$ and $\|d\nu/d\pi\|_{L^r(\pi)}$ such that for all $N \geq 1$,*

$$\mathbb{E}_\nu\left(\mathcal{T}_p(\mu_N, \pi)\right) \leq C \begin{cases} N^{-1/2} + N^{-(q_r - p)/q_r} & \text{if } p > d_r/2 \text{ and } q_r \neq 2p, \\ N^{-1/2}\log(1+N) + N^{-(q_r-p)/q_r} & \text{if } p = d_r/2 \text{ and } q_r \neq 2p, \\ N^{-p/d_r} + N^{-(q_r-p)/q_r} & \text{if } p \in (0, d_r/2) \text{ and } q_r \neq d_r/(d_r - p). \end{cases}$$

References

[1] N. Fournier, A. Guillin. On the rate of convergence in Wasserstein distance of the empirical measure. Probab. Theory Related Fields 162 (2015), 707-738.